The Cost of a Bad Hire: On the Optimality of Pay to
Quit Programs*

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Abstract

Contracts that compensate workers if they choose to leave a firm – known as Pay to
Quit programs – have been implemented by both established and new companies. One the one hand, such programs may help firms avoid the cost of low quality employees. On the other hand, resources used for these programs are a net loss for the firm and could be used on productive employees instead. In this paper, we examine when it is optimal for firms to offer employment contracts with Pay to Quit options. We propose a model to study the optimal employment contract under both moral hazard – the firm must incentivize employee effort – and adverse selection – the firm must select out a bad fit in order to access a better outside option. We derive the conditions under which offering a payment to quit is optimal and provide testable empirical implications.

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1 Introduction

An increasingly observed practice among both small and large companies has been to incentivize the early removal of potentially “bad fits” by offering contracts that compensate employees who decide to quit – called Pay to Quit programs. The online retailer Zappos famously introduced such contracts in 2007, allowing its employees to exercise the Pay to Quit option at the end of their training period. Similar programs were subsequently adopted by Amazon in 2014 and by some Silicon Valley start-ups.¹

On the one hand, these programs seem ingenious. By paying employees to separate and to do so early, firms can avoid the costs of a low quality and potentially disruptive employee. On the other hand, the underpinning logic of this screening mechanism is far from clear. Payments to quit represent a net loss for the firm – they are made precisely to non-employees – and these resources may be more efficiently used incentivizing productive behavior among continuing employees. Moreover, there is nothing stopping the best employees, rather than the worst, from taking the payments and leaving the firm.

It remains an open question, therefore, whether Pay to Quit programs represent a contracting innovation or whether they are a mere marketing gimmick. The objective of this paper is to examine the underlying logic of Pay to Quit programs and to answer this question. Our conclusion is that these programs can represent optimal contracts, although only in particular circumstances. We identify and fully characterize these circumstances and show that they depend on a condition we refer to as Relative Value of Employment, RVE for short, for both employers and employees. This new and economically meaningful condition captures the change when moving from the best fit employees to the worst fit ones in how much being employed by the firm is valued relative to the wage offered. Surprisingly, for Pay to Quit to be optimal, there must exist a disconnect between the RVE of the employer and specifically those employees the firm wishes to keep rather than drive away. Thus, the foundation for Pay to Quit programs is indeed employee “fit,” but not in the sense that they align closely

¹CBS News, “Why Amazon pays employees $5,000 to quit,” April 11, 2014; CNBC, “This company will pay you a bonus to quit your job,” September 30, 2016.
with the employer but rather that they oppose along the RVE dimension.

In the model we develop, we operationalize the importance for the firm of selecting the right fit by supposing that it has an outside option should an employee separate—the firm may employ an alternative worker or pursue an alternative project—and, thus, the firm derives negative relative utility should a low quality type remain employed. We then analyze in this context an otherwise standard contracting problem in the presence of adverse selection—some employees have a lower cost for the firm than others—and moral hazard—the firm must incentivize effort.

Specifically, we study an environment in which a firm faces both high ability and low ability employees. Higher ability employees are more valuable to the firm since they have a lower cost of supplying effort for the firm’s project. Yet, neither employee ability nor effort can be directly observed by the firm. Due to the availability of an outside option, the firm would like to incentivize the low ability employees to quit; however, the incentives offered must not be so tempting that high ability employees also choose to take them. Moreover, for those employees who stay, the firm must incentivize effort on the project. To achieve these goals, the firm can vary the wage based on the project outcome, it can vary the probability of continued employment—for example, by assigning the employee to a specific unit within the firm, where different units face different probabilities of being dismissed—or it may allow the employee to seek the payment to quit.

The main insights of the paper emerge from the following logic. If employee ability were perfectly observable, the firm would remove the low ability employee without any compensation and keep the high ability employee; however, when ability is not observable, the low ability worker does not voluntarily select into such a contract. Thus, if no payment to quit is possible, then the firm, unable to outright remove the low ability employee, could try to offer a separate contract for each employee type. Since one contract cannot be superior to the other on all dimensions—or else the second contract would not be chosen—, one of the two contracts must specify a higher probability of continued employment—placement in a better unit within the firm—, along with a lower wage in case of project completion. Since
workers voluntarily select their desired contract, the firm must design the contract menu such that each employee selects the contract intended for his type.

The employees’ contract choice depends on how each of them values continued employment relative to wage. Therefore, inferring which contract will be chosen by each type reduces to examining the analogue to the standard single crossing condition in this contracting environment. We derive this analogue and relate it to a measure of the Relative Value of Employment (RVE). The high (low) ability employee voluntarily takes the contract with the higher (lower) probability of continued employment and the lower (higher) wage if the high ability type values the gain from a higher probability of continued employment relative to the loss from a lower wage more than (less than) the low ability type, i.e., the marginal rate of substitution of wage for probability of continued employment (the RVE)\(^2\) is decreasing (increasing) in the employee type. Hence, the set of contracts implementable without a payment to quit is determined by how the RVE changes with the employee’s type.

Among the implementable contracts, the firm would like to select the one that gives it the highest gains. We show that the firm’s problem of choosing the best contract reduces to studying the RVE condition for the firm – how firm’s relative valuation of retaining an employee (to address adverse selection) to offering wage compensation (to address moral hazard) changes when it faces an employee it would like to keep versus an employee who is a “bad fit.” We show that when the RVE of the firm and the employee are aligned in a key dimension – they both increase or both decrease in response to a change in employee type – then the firm’s preferred contract is implementable without a payment to quit. If the RVE of the firm and the employee move in different directions, then the firm’s preferred contract is not implementable without a payment to quit – the contract would not be incentive compatible. In this case, adding a payment to quit to the contract designed for the “bad fit” makes it possible to offer the firm’s preferred contract menu. The logic for this result is that a payment to quit gives the firm an additional dimension of contracting freedom.

Our main result shows that all three main components of the model – moral hazard,

\(^2\)I.e., how much decline in continuation probability the employee is willing to accept in exchange for one unit increase in wage.
adverse selection, outside option – are necessary for a Pay to Quit program to be optimal. Without the firm’s outside option, separation would not be desirable for the firm. Without adverse selection, a payment to quit would not be used since it does not incentivize production. Without moral hazard and with both good and bad fits employed, a firm’s optimal contract would specify a payment to the “bad type” in the case of project failure, rather than a payment to quit, since project failure is the best indicator of employee ability. Thus, without all three components, a payment to quit would be suboptimal. Moreover, we show that firm-specific fit is the “best case scenario” under which Pay to Quit programs may be optimal. We provide an extension to the model that adds type-dependent outside values for the employees, to reflect genuine productivity differences, and show that, in such cases, offering a payment to quit may backfire and result in the high ability types selecting out.

Our findings inform for which firms and in which situations Pay to Quit programs will be effective. Informal descriptions of the program emphasize the importance of fit.3 Our formal results reinforce this viewpoint, yet show that what is actually important is the misalignment of fit captured by the RVE measure. It is necessary that the “best fit” employees value compensation relatively more than actually working for the firm than do “bad fits.” Therefore, the model produces testable empirical implications as to which firms and industries may use such incentive schemes in order to effectively select employees.

**Related Literature** Agency theory has extensively explored the optimal design of contracts with either moral hazard or adverse selection;4 however, the case in which both moral hazard and adverse selection are present is less well understood. Chade and Swinkels (2016) analyze the general principal-agent problem with both moral hazard and adverse selection. The main differences between the environment they analyze and our paper are the existence of the firm’s outside option, and that we allow the firm to also target the probability with

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which an employee stays on to work on the project.\footnote{In Section 4.5, we discuss in greater detail the relationship between our results and their work.} Gottlieb and Moreira (2014) also analyze a principal-agent problem with two types of agents who have private information about their type and about the distribution of outcomes conditional on whether effort was exerted. We consider this same setting, but add another component to the principal’s problem: the firm’s outside option if the current employee is not retained. While Gottlieb and Moreira (2014) obtain the result that all agents are offered a single contract, we show that with an outside option, the optimal contract can in fact be differentiated for each type, and it may feature a payment to leave.

There is a rich and growing literature on dynamic contracts with both moral hazard and adverse selection (Strulovici, 2011; Garrett and Pavan, 2012; Halac et al., 2016); while the focus in this literature is on the evolution of the relationship between the firm and the employee once work on the project happens, our paper examines instead the problem of selecting out a bad type employee ex ante, in environments in which the cost of not immediately removing a “bad hire” is high. To focus on ex ante selection, we analyze the environment with one period of production.

Our paper is also related to the literature on screening in the presence of both moral hazard and adverse selection (Chiu and Karni, 1998; De Meza and Webb, 2001; Jullien et al., 2007), as we examine optimal contracts in environments in which it is valuable to select out low ability types. While this literature considers a similar framework, we add to it the outside option for the firm if it does not retain the employee.

In the applied theory literature, our topic is related to the literature that examines the use of severance pay in CEO compensation (Almazan and Suarez, 2003; Manso, 2011; Inderst and Mueller, 2010). These works have proposed models that focus the optimality of severance payments after a CEO has worked on a project. We add a new dimension to this literature. Our model could be applied to a board’s problem of whether to use severance payments in order for a bad CEO to select out before a major new project, for instance a large acquisition or entering a new market – thus, before a CEO exerts effort on the project.
Our topic is also related to the literature on incentives provision for downsizing the public sector. Jeon and Laffont (1999) consider the optimal contracts for inducing low ability public employees to voluntarily leave a public sector job. While their model only considers the adverse selection problem, we examine optimal contracts with both adverse selection and moral hazard.\(^6\)

The rest of the paper is organized as follows. Section 2 outlines the features of Pay to Quit programs. Section 3 describes the game. It also introduces the analogue to the standard single crossing condition in this environment, namely log-submodularity/ supermodularity, and explains how this notion is related to our notion of RVE. Section 4 studies the optimal employment contract, and Section 5 illustrates it by numerical examples. Section 6 presents several extensions that show the robustness of the optimal contract to introducing outside options for agents, multiple project outcomes, an endogenous outside option for the firm, and multiple agent types. Section 7 discusses empirical implications, and Section 8 concludes. The Appendix contains the main proofs.\(^7\)

2 Primer on Pay to Quit Programs

The first company to introduce a Pay to Quit program was the online retailer Zappos in 2007. The program was first aimed only at the consumer service employees, and it was later extended to all employees. The program operated as follows. Each employee started their employment with an intense four-week training course. After a week of training, Zappos made them the Pay to Quit offer: “Quit today, or anytime before the training is over, and we’ll give you a $2,000 bonus.” The intent, as summarized by the company’s CEO, was to incentivize the early selection of workers who discover during training that they are not good fits for the company. Overall, approximately 3% of the employees took the offer.\(^8\) In 2014,

\(^6\)An alternative approach to the topic of selecting employees given firm-specific fit is provided by the literature on optimal contracts with intrinsically motivated employees (Murdock, 2002; Benabou and Tirole, 2003; Besley and Ghatak, 2005). We do not assume intrinsic motivation, and instead derive the optimal contract when the firm-specific fit only affects the employee’s cost of effort.

\(^7\)The Online Appendix contains further proofs and extensions of the model.

\(^8\)According to statements by Zappos CEO Tony Hsieh, cited by Staffing Talk in January 2013.
online retailer Amazon introduced a similar program. The monetary compensation in its case was $5,000, and the offer was made annually, not just at the end of the training period. Other companies have also adopted variations of the Pay to Quit program. For example, one Silicon Valley startup offered a bonus equal to 10% of salary to the employees who decided to quit.\footnote{CNBC, “This company will pay you a bonus to quit your job,” September 30, 2016.} Nevertheless, Pay to Quit programs remain relatively new, and, while increasingly used by employers, they are yet to gain widespread adoption.

A related set of programs used to select out bad fits are programs that allow employees to choose a specific team or career track inside the company – a form of paid internal selection without anyone leaving the firm. Depending on their team membership, employees face different probabilities of working on projects with high monetary or career progressing rewards. For example, several corporate law firms have introduced early tracking within the firm, with a partner track as well as a “permanent associate” track. The latter offers a reduced probability of career advancement and lower compensation, but in exchange provides the employees with more job security and a less demanding schedule.\footnote{The New York Times, “At Well-Paying Law Firms, a Low-Paid Corner,” May 23, 2011.}

In the following section, we introduce a model that captures the main elements implied by the description of these programs. We allow for a situation where, at the end of an initial training period, employees gain more information about their fit than the firm does. The contract, which may contain a payment to quit, is offered at the end of the training period, before any work is done for the firm. Instead of just considering the choice between immediately quitting and staying on to work on a high reward project, we introduce more flexibility in the contracts, so as to also capture the situations described above as intra-firm selection. Specifically, we allow contracts to give the employee a probability of working on the high rewards project, depending on which team (or track) he selects in. The case in which this probability is either 0 or 1 corresponds to the pure Pay to Quit programs. In the next section, we further describe the environment and the contract space.
3 The Model

We consider an environment with two players: a principal (the firm) and an agent (the worker). The agent has one of two possible types, \( \theta \in \{H, L\} \).\(^{11}\) This type is the agent’s private information, while the principal has prior belief \( \mu_\theta \) that the agent is of type \( \theta \).

The agent is hired by the principal to work on a project, the outcome of which depends on the agent’s private effort. The agent of type \( \theta \) has a cost of effort \( c(\theta, e) \) for \( e \in \mathbb{R}_+ \), which satisfies

\[
0 \leq c(H, e) \leq c(L, e), \quad c_e, c_{ee}, c_{eee} \geq 0.\(^{12}\)
\]

Moreover, type \( H \) has lower marginal cost of effort than type \( L \):

\[
c_e(H, e) \leq c_e(L, e) \quad \text{for each } e \in \mathbb{R}_+.
\]

The set of possible outcomes is given by \( Y = \{0, 1\} \), where \( y = 1 \) denotes a project success and \( y = 0 \) denotes a failure.\(^{13}\) The probability of a success is \( q(e_\theta) \), where

\[
q(e_\theta) = q_0 + q_1 e_\theta, \quad \text{with } q_0, q_1 \in \mathbb{R}.\(^{14}\)
\]

By the revelation principle, we can focus on the contracts in which the agent truthfully declares his type. Depending on the declared type \( \theta \), the principal offers an employment contract \( C_\theta \). A contract can specify three elements: a probability \( p_\theta \) of being retained to work on the project, a wage \( w_\theta(y) \) contingent of the realized output \( y \in Y \), if the agent is retained on the project, and a payment \( w_\theta(\emptyset) \) in case of no retention. The elements of the contract are further described and motivated below.

First, the probability of being retained to work on the project is motivated by the description in Section 2. To fully capture an environment akin to our application, we can

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\(^{11}\) In Section 6.4, we provide an extension to more than two types.

\(^{12}\) The subscripts \( e \) denote derivatives with respect to effort.

\(^{13}\) See Section 6.2 for the case with more than binary outcomes.

\(^{14}\) Note that, as long as \( q \) is concave, we can always re-measure effort such that linearity is obtained for convex cost \( c \).
add the following extensive form to our framework, such that, once matched with a firm, the employee undergoes a training period. At the beginning of the training period, neither the employee nor the firm know the employee’s type. During the training period, however, the employee learns his type, while the firm forms belief $\mu_\theta$ about his type. Yet, nothing is produced and no effort is exerted during training.\footnote{Since the training period has no cost, the analysis is independent of their prior beliefs about the type.} At the end of the training period, the employee is given a set of continuation options. The menu of options includes joining a specific team within the firm, and it may include the choice to outright quit. A team for type $\theta$ has a probability $p_\theta$ of getting to work on the project, and a probability $1 - p_\theta$ of being dismissed or given other tasks that do not involve high-powered incentives. An option to outright quit is a contract with $p_\theta = 0$.

Second, the wage $w_\theta(y)$ reflects the ability of the principal to incentivize effort and compensate the agent who worked on the project, based on the success of the project. Finally, if the agent is not retained, he leaves right away and is paid $w_\theta(\emptyset)$. He does not start work on the project, so he does not exert any effort.

**Timing of Actions.** The game can be summarized in the following timing of actions:

1. The agent declares his type $\hat{\theta}$. On the equilibrium path, he reports his true type: $\hat{\theta} = \theta$.

2a. With probability $p_\theta$, the principal retains the agent for the project. After retention, the agent chooses private effort $e_\theta$ and is paid $w_\theta(y)$ given the observable outcome $y$. This is the end of the game.

2b. With probability $1 - p_\theta$, the principal does not retain the agent. She pays $w_\theta(\emptyset)$ to the agent. The principal obtains an exogenous outside option with value $W$.

For notational convenience, we also define

$$w_\theta \equiv w_\theta(y = 1) - w_\theta(y = 0),$$
the relative reward received by a retained agent \( \theta \) in case of a success rather than a failure, and

\[
v_\theta = p_\theta \cdot w_\theta(y = 0) + (1 - p_\theta) \cdot w_\theta(\emptyset),
\]

the expected base payment if nothing is produced – either because the project fails or because the agent is not retained. This is what we call a “fixed payment” – the payment not contingent on anything being produced.

As shown above, in the full model we assume that the agent’s type \( \theta \) is not observable (adverse selection), and that effort \( e_\theta \) is also not observable (moral hazard). One may wonder whether the application motivating the model warrants considering both adverse selection and moral hazard, or if instead it is not just a story of adverse selection. To address this, in Section 4.1 we present the analysis of the case with adverse selection only, where effort is inconsequential for the project success. We show that moral hazard considerations are necessary for an economically meaningful analysis of contracts with payments to quit. Finally, we also assume limited liability: \( w_\theta(y) \geq 0 \) and \( w_\theta(\emptyset) \geq 0 \) \( \forall \theta, y \).\(^{16}\)

**Payoffs.** Given the risk neutrality, the total expected payoff for agent \( \theta \) from contract \( C_\theta \) is given by

\[
p_\theta \cdot V(\theta, w_\theta) + v_\theta,
\]

where

\[
V(\theta, w_\theta) \equiv q(e(\theta, w_\theta)) \cdot w_\theta - c(\theta, e(\theta, w_\theta))
\]
denotes the payoff given the optimal effort with reward \( w_\theta \). The equilibrium effort \( e(\theta, w_\theta) \) for type \( \theta \) is determined by

\[
e(\theta, w_\theta) = \arg \max_e q(e) \cdot w_\theta - c(\theta, e).
\]

We assume that type \( H \) brings strictly higher profit to the principal than type \( L \) for the same wage:

\(^{16}\)Otherwise, “selling a project” to the agent, with a price accepted only by the high type is optimal.
Assumption 1 The cost \( c(\theta, e) \) is continuously differentiable with effort \( e \in \left[ 0, \frac{1-q_0}{q_1} \right] \), 
\[
\lim_{e \to 0} c_e(L, e) = 0, \lim_{e \to \frac{1-q_0}{q_1}} c_e(H, e) \geq q_1, \text{ and } c_e(H, e) - c_e(L, e) < 0 \text{ for each } e > 0.
\]

Assumption 1 provides that sufficient conditions such that \( \forall w, e(H, w) > e(L, w) \), and for type \( H \), \( e(H, w) < 1 \), since it is suboptimal for the principal to offer \( w_{\theta} \geq 1 \).

Assumption 2 The following condition holds: \( W > \max_w \pi(L, w) \).

Assumption 2 allows us to focus on the case in which selecting the “right fit” is of high importance for the principal. Specifically, with full information over the agents’ types, the principal prefers the outside option over hiring type \( L \).

3.1 The Principal’s Problem

Upon retaining an agent of type \( \theta \), the principal obtains expected payoff
\[
\pi(\theta, w_{\theta}) \equiv q(e(\theta, w_{\theta}))(1 - w_{\theta}).
\]

Defining the social welfare function as \( S(\theta, w_{\theta}) \equiv q(e(\theta, w_{\theta})) - c(\theta, e(\theta, w_{\theta})) \), we can write
\[
\pi(\theta, w_{\theta}) = S(\theta, w_{\theta}) - V(\theta, w_{\theta}). \tag{3}
\]

We analyze the optimal contract offered by the principal in order to maximize her expected profit, under the constraint that the ex-ante value for each agent \( \theta \) from contract \( C_{\theta} \) is no less than \( C_{\hat{\theta}} \) for each \( \hat{\theta} \neq \theta \). Given \( W \), her problem is characterized by the following program:
\[
\tilde{J}(W) = \max_{(p_H, p_L, w_H, w_L, v_H, v_L)} \mu_H \left[ p_H \pi(H, w_H) + (1 - p_H)W - v_H \right] + \mu_L \left[ p_L \pi(L, w_L) + (1 - p_L)W - v_L \right], \tag{4}
\]

\[17\text{In Appendix G, we provide the analysis of the case in which Assumption 2 does not hold.}\]
subject to

\[ p_H V(H, w_H) + v_H \geq p_L V(H, w_L) + v_L; \quad (IC_H) \]
\[ p_L V(L, w_L) + v_L \geq p_H V(L, w_H) + v_H. \quad (IC_L) \]

The principal maximizes her payoff \( J(W) \) subject to the two incentive compatibility constraints. Constraint \( IC_H \) ensures that the type \( H \) agent prefers the contract \((p_H, w_H, v_H)\) to the contract designed for the type \( L \) agent, \((p_L, w_L, v_L)\). Similarly, constraint \( IC_L \) ensures that the type \( L \) agent prefers the contract \((p_L, w_L, v_L)\) to the contract designed for the type \( H \) agent, \((p_H, w_H, v_H)\). We present the problem and results by assuming that \( W \) is exogenous, \( W \in (0, W^{MAX}) \), where \( W^{MAX} = \max_{w_H} \pi(H, w_H) \).\(^{19}\) In Section 6.3, we endogenize \( W \).

Throughout the paper, we analyze the non-trivial case in which the principal hires at least one of the agents with positive probability, so \( \max \{p_H, p_L\} > 0. \)

### 3.2 A Useful Condition

As mentioned in the introduction, the key condition for \( v_L > 0 \) will be the co-movement between the principal’s and agent’s relative value of employment (RVE). To characterize when there is co-movement, we make use of the following property.

**Definition 1** A function \( F(\theta, w) \) is **log-supermodular** (**log-submodular**) if the following property holds:

\[
F(\theta, w) F(\theta', w') - F(\theta', w) F(\theta, w') \geq (\leq) 0
\]
if and only if \((\theta - \theta')(w - w') \geq (\leq) 0.\]

If \( \log F(\theta, w) \) is twice-differentiable, then it is log-supermodular (log-submodular) whenever

\(^{18}\)The definition of \( V(\theta, w) \) and \( \pi(\theta, w) \) implies (2).

\(^{19}\)If \( W \geq W^{MAX} \), not hiring any candidate is trivially optimal.

\(^{20}\)The guess is verified if the optimal value with this guess is greater than \( W \). Otherwise, \( p_H = p_L = v_H = v_L = 0 \) is optimal.
We say a function is regular if, given \( \theta, \theta' \in \{L, H\} \), it is globally log-supermodular or log-submodular for each \( w, w' \in [0, 1] \) with \( F(\theta, w), F(\theta, w'), F(\theta', w), F(\theta', w') \geq 0 \).

Intuitively, log-supermodularity implies that an increase in \( w \) is relatively more valuable when \( \theta \) is higher. Applied to \( V(\theta, w) \), log-supermodularity means that, all else constant, an increase in the reward \( w \) leads to a relatively higher increase in type \( H \)'s payoff. If \( S(\theta, w) \) is also log-supermodular, then an increase in \( w \) increases the social welfare upon retention relatively more when the agent is of type \( H \).

To show why the property of log-supermodularity/submodularity will be useful in analyzing our problem, we provide the following intuition. Consider the case in which no fixed payment is offered, so \( v_H = v_L = 0 \). In this case, the principal offers a contract that features a reward \( w \) and a probability of retention \( p \). We can therefore represent each contract on the two dimensional space of \((p, w)\). For any arbitrary contract \( C_H \), consider the set of contracts \( C \) that would make type \( \theta \) indifferent between \( C_H \) and \( C \). On the indifference curve obtained in this way, the marginal rate of substitution of \( w \) for \( p \) satisfies

\[
\frac{dp}{dw} = \frac{V_w(\theta, w)}{V(\theta, w)}.
\]

The log-submodularity of \( V(\theta, w) \) implies that \( dp/dw \) is decreasing in \( \theta \) – marginal rate of substitution of the wage for the probability of retention is decreasing in the agent’s type.

We refer to this as decreasing RVE Type \( H \) is willing to sacrifice more monetary rewards in exchange for the increase in retention probability – he values higher retention probabilities relative to higher rewards – compared to type \( L \). This is illustrated in Figure 1. The intersection of the indifference curves is \( C_H \) and the shaded area is the set of \( C_L \) that satisfies \((IC_H)\) and \((IC_L)\) with \( v_L = 0 \).

The same analysis as above also applies to the principal’s payoff, if we consider the case in which the agent receives a fixed payoff. Given transferable utility, the principal maximizes

\[
\Delta \frac{d^2}{dbdw} \log F(\theta, w) \geq (\leq)0.
\]
the net social welfare created by the agent $p \left( S(\theta, w) - W \right)$.

Applying the above intuition, the log-submodularity of $S(\theta, w) - W$ implies that the principal is more willing to give up offering high rewards in exchange for increasing retention, i.e., she values higher retention probabilities relative to higher rewards when faced with type $H$, compared to the case when faced with type $L$. This condition is equivalent to the principal’s marginal rate of substitution of the wage for the probability of retention decreasing in the agent’s type. Again, we refer to this as decreasing RVE.

Hence, the co-movement of RVE – when the RVE of the principal and the agent both decrease or both increase – happens when both $V(\theta, w)$ and $S(\theta, w) - W$ are log-submodular or both of them are log-supermodular. The RVE counter-movement happens when one expression is log-submodular and the other is log-supermodular.

In order to streamline the analysis and highlight the intuition behind the main result, we present the results for the case in which $V(\theta, w)$ is globally log-submodular and $S(\theta, w) - W$ is regular. In Appendix F, we provide the complementary analysis.

**Remark 1** There exist cost functions $c(\theta, e)$ such that $V(\theta, w)$ is log-submodular and $S(\theta, w) - W$ is regular. If constraint $IC_\theta$ is binding, so the rent to the agent of type $\theta$ is “fixed” given contract $C_{\theta'}$ offered to type $\theta' \neq \theta$.\footnote{This occurs if contraint $IC_\theta$ is binding, so the rent to the agent of type $\theta$ is “fixed” given contract $C_{\theta'}$ offered to type $\theta' \neq \theta$.}
$W$ is regular.

An example of a cost function that leads to $V(\theta, w)$ log-submodular and $S(\theta, w) - W$ is regular is given in Section 5.

**Assumption 3** The cost function $c(\theta, e)$ has a form that leads to $V(\theta, w)$ log-submodular and $S(\theta, w) - W$ regular.

Under this assumption, we proceed to analyze the optimal employment contract.

## 4 The Optimal Contract

### 4.1 Benchmark with Adverse Selection Only

We begin with a simplified version of the model, in which effort is inconsequential to the project success. Therefore, the only concern for the principal is that of adverse selection. By providing this benchmark analysis, we articulate why the problem would be incomplete without considering moral hazard as well.

Suppose that type $\theta \in \{H, L\}$ has the probability $q_\theta$ of achieving outcome $y = 1$. This probability is independent of effort and $q_H > q_L$. The rest of the setup is as described above.

First, we notice that, if the agent has no outside option, then the principal’s problem is to choose $\{p_\theta, w_\theta\}_{\theta \in \{L, H\}}$ to solve the following program:

$$\max p_H \cdot (q_H \cdot (1 - w_H) - W) - v_H + p_L \cdot (q_L \cdot (1 - w_L) - W) - v_L$$

subject to

$$p_H \cdot q_H \cdot w_H + v_H \geq p_L \cdot q_H \cdot w_L + v_L; \quad (6)$$

$$p_H \cdot q_L \cdot w_H + v_H \leq p_L \cdot q_L \cdot w_L + v_L. \quad (7)$$

The constraints represent the incentive compatibility constraints for types $H$ and $L$, respectively. In this problem, it follows immediately that $p_H = 1$ and $p_L = w_H = v_H = v_L = 0.$
The optimal contract for the principal is to retain the $H$ type agent and make no payments to the agent.

A richer analysis of the case with only adverse selection would require us to assume that the agent has an outside option. We perform this analysis in the Appendix, and show that it leads to a result that is intuitive, but at odds with our application. With adverse selection only, if both types are employed, then the project success ($y = 1$) is a stronger signal of a high type of agent. Therefore, it is preferable to compensate the $H$ type only after outcome $y = 1$. Conversely, the $L$ type should only be compensated when the project fails. This stark solution to the problem highlights the importance of adding moral hazard considerations if the model is to lend itself well to an economically meaningful analysis of Pay to Quit programs.

4.2 Preliminaries

We now switch to analyzing the full model, with both adverse selection and moral hazard. As a first step, we derive several properties of objective (4), that allow us to characterize the optimal contract. First, Assumption 2 implies that constraint $IC_L$ holds with equality, since otherwise the principal would like to reduce $p_L$. Hence, the principal faces the threat of type $L$ imitating type $H$, and she must therefore adjust the menu of contracts to dissuade type $L$ from choosing the contract designed for type $H$.

Next, we also show that the type $H$ agent is offered no fixed payment, and at least one type of agent is always retained for the project.

Lemma 1 The optimal contract has the property that no fixed payment is offered to the type $H$ agent ($v_H = 0$), and one of the agent types is always retained (either $p_L = 1$ or $p_H = 1$).


The principal only rewards the $H$ type in case of the project’s success. Intuitively, increasing the reward increases the effort provided by the agent. Since the agent is risk-neutral, the principal can incentivize the highest effort from type $H$ by loading the entire
reward on the positive outcome. Moreover, this change makes type L less willing to claim to be of type H since he has to pay a higher effort cost for a good outcome. Finally, at least one of the agents is always retained. Otherwise, increasing $p$ and $v$ proportionally would proportionally increase the principal’s payoff relative to the outside option $W$.

Next, we establish that the incentive compatibility constraint for type H must hold with equality whenever $v_L > 0$.

**Lemma 2** In the optimal contract, if the type L agent is offered a fixed payment ($v_L > 0$), then constraints $IC_L$ and $IC_H$ bind.

**Proof.** In Appendix A.4. ■

The fixed payment $v_L > 0$ amounts to a cost for the principal without any benefit in terms of output. With constraint $IC_L$ binding, the principal can reduce $v_L$ and increase $w_L$, keeping the payoff to type L constant. This is feasible as long as type H still prefers the contract designed for his type, i.e., up until constraint $IC_H$ binds.

Finally, we show that, even with a fixed payment, the principal cannot offer both a lower reward and a lower retention probability to type L.

**Lemma 3** It is the case that “$w_H \geq w_L$ and $p_H \leq p_L$” or “$w_H \leq w_L$ and $p_H \geq p_L$.”

**Proof.** In Appendix A.5. ■

Without a fixed payment, offering type L both a lower reward and a lower retention probability is not incentive compatible. With a fixed payment, the previous lemma implies constraints $IC_H$ and $IC_L$ bind, which excludes any other combination of wages and retention probabilities than the ones stated in the lemma.

Given the above preliminaries, we proceed to solve for the optimal contract.

### 4.3 High Outside Option

First, if type L is very unproductive compared to the value of the outside option, then the principal does not retain this type:
Proposition 1 (Retention of type H only) If $W \geq S(L, 1)$, then only the type H agent is retained in the optimal contract: $v_L > 0$, $p_L = 0$, $p_H = 1$. Conversely, if $W < S(L, 1)$, then type L is retained with positive probability ($p_L > 0$).


If $W \geq S(L, 1)$, then type L is not retained. He is instead offered a fixed payment that makes his contract as appealing as taking the contract designed for type H. By not retaining type L, the principal loses at most $S(L, 1)$ and she gains $W$ from the outside option. Conversely, if $W < S(L, 1)$, then the principal can gain by retaining type L. To see this, consider starting from $p_L = 0$ and $w_L = 1$ and increasing $p_L$ by a small $\varepsilon > 0$. The principal can then reduce $v_L$ by $\varepsilon V(L, 1) = \varepsilon S(L, 1)$ and keep type L indifferent.\footnote{Type H’s incentive compatibility is satisfied for a small $\varepsilon$.} Since the principal has to give up the outside option when she retains type L, her net gain is $\varepsilon (S(L, 1) - W)$. Therefore, this would be a gainful deviation for the principal.

Next, we consider the case in which $W$ is sufficiently low, so that type L is retained with positive probability.

4.4 Moderate Outside Option

Lemma 3 implies that the principal faces a trade-off between solving the moral hazard problem (by offering $w_H \geq w_L$) and solving the adverse selection (by offering $p_H \geq p_L$). The following result shows how this trade-off is resolved, and when the fixed payment is part of the optimal contract:

Proposition 2 Suppose $V(\theta, w)$ is log-submodular. When $W < S(L, 1)$, the optimal contract has the following properties:

1. (RVE Co-Movement) If $S(\theta, w) - W$ is also log-submodular, then $v_L = 0$, $w_L \geq w_H$, and $1 = p_H \geq p_L > 0$.  

Type H’s incentive compatibility is satisfied for a small $\varepsilon$.  

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2. (RVE Counter-Movement) If $S(\theta, w) - W$ is log-supermodular, then $v_L \geq 0$, $w_H \geq w_L$, and $1 = p_L \geq p_H > 0$. Moreover, a fixed payment is optimal ($v_L > 0$) if and only if $w_H \neq w_L$.

**Proof.** In Appendix A.8. 

The focus of this central result is on the properties of $V(\theta, w)$ and $S(\theta, w) - W$. The value $V(\theta, w)$ captures the agent’s payoff from working. With the agents’ incentive compatibility constraint binding for the low ability type, the total payoff for the agent of type $L$ is “fixed” given contract $C_H$ offered to type $H$. Then, the principal maximizes the net social welfare created by the type $L$ agent, $p (S(\theta, w) - W)$, minus the fixed payoff to the agent. Therefore, studying the principal’s maximization problem reduces to examining $S(\theta, w) - W$.

Analytically, the result of Proposition 2 hinges on the relationship between the log-supermodularity (log-submodularity) properties of $S(\theta, w) - W$ and $V(\theta, w)$. First, $S(\theta, w) - W$ being log-submodular (log-supermodular) determines $w_L \geq w_H$ ($w_H \geq w_L$) in the optimal contract. Starting from a set of contracts with $w_L \neq w_H$, the principal has two possible incentive compatible deviations that she could undertake: offer all agents the contract designed for type $H$, or offer all agents the contract designed for type $L$. If the principal offers all agents the contract designed for type $H$, then the principal gains over the original set of contracts if this new contract offers her a higher payoff from type $L$, i.e.,

$$p_L (S(L, w_L) - V(L, w_L)) + (1 - p_L) W - v_L < p_H (S(L, w_H) - V(L, w_H)) + (1 - p_H) W.$$ 

Given constraint $IC_L$, such a deviation is not profitable only if

$$p_L (S(L, w_L) - W) \geq p_H (S(L, w_H) - W).$$  \hspace{1cm} (8)
Similarly, the principal’s deviation to offer both types the contract designed for the type $L$ agent is not profitable only if

$$p_L (S (H, w_L) - W) \leq p_H (S (H, w_H) - W).$$

(9)

Combining (8) and (9) delivers the result that the log-submodularity of $S (\theta, w) - W$ is equivalent to $w_L \geq w_H$, and the log-supermodularity of $S (\theta, w) - W$ is equivalent to $w_H \geq w_L$. Intuitively, log-submodularity of $S (\theta, w) - W$ means that an increase in $w$ granted to type $L$ is relatively more valuable for the excess social welfare than an increase in $w$ granted to type $H$. That is, the log-submodularity implies that the principal puts more weight on solving the adverse selection problem, by offering $w_H \leq w_L$ and $p_H \geq p_L$. Symmetrically, the log-supermodularity implies that the principal puts more weight on solving the moral hazard problem by offering $w_H \geq w_L$ and $p_H \leq p_L$.

Second, the log-submodularity of $V (\theta, w)$ determines whether a fixed payment will be used in the optimal contract. Consider the case in which $S (\theta, w) - W$ is log-submodular, so that $w_L \geq w_H$, and $p_H \geq p_L$, by Lemma 3. This implies that type $H$ is compensated for the reduced reward $w_H$ by being offered a higher probability of retention $p_H$. If $V (\theta, w)$ is log-submodular, then such an increase in $p_H$ is relatively more valuable to type $H$ than to type $L$. Thus, in this case, the principal’s preferred contract menu is incentive compatible, and no fixed payment $v_L \geq 0$ is necessary. If $S (\theta, w) - W$ is log-supermodular, $w_H \geq w_L$ and $p_H \leq p_L$. With $V (\theta, w)$ log-submodular, the decrease in $p_H$ is relatively more beneficial for type $L$ than for type $H$. Then, without a fixed payment, type $L$ is better off by claiming to be type $H$; however, the fixed payment $v_L \geq 0$ can be added to type $L$’s contract in order for him to become indifferent between the two contracts. Thus, the fixed payment can be used to obtain an incentive compatible contract menu.

The intuition for the result is as follows. If both $S (\theta, w) - W$ and $V (\theta, w)$ are log-submodular, then the principal’s and the agent’s RVE co-move – they are both decreasing in the agent’s type. This means that the principal values solving the adverse selection problem by increasing $p_H$ relatively more than solving the moral hazard problem by increasing $w_H$. 

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and the type $H$ agent values retention – the action that solves adverse selection – over wage – the action that solves the moral hazard problem – relatively more compared to type $L$. When $S(\theta, w) - W$ is log-supermodular, the RVE of the principal and of the agent counter-move. The principal now values solving the moral hazard problem by increasing $w_H$ relatively more than solving the adverse selection problem. In this case, the fixed payment $v_L \geq 0$ is required in order to ensure incentive compatibility of the optimal contract.

When is a Fixed Payment Optimal?

Proposition 2 shows that a fixed payment to the low type agent is optimal when there is counter-movement in the RVE of the principal and the agent. In the following Corollary, we provide specific conditions for when RVE counter-movement is more likely to occur.

**Corollary 1** A fixed payment for the low ability worker is offered in the optimal contract if $e_\theta(\theta, w)$ and $S_\theta(\theta, w)$ are sufficiently small, and $e_{w,\theta}(\theta, w)$ and $V_\theta(\theta, w)$ are sufficiently large.\(^{24}\)

**Proof.** In Appendix A.9.

The conditions for the RVE counter-movement capture the requirements for obtaining a log-submodular $V(\theta, w)$ and a log-supermodular $S(\theta, w) - W$. First, for the agent’s RVE to be decreasing in type, $V(\theta, w)$ must be log-submodular. Condition 5 requires a sufficiently small $e_\theta(\theta, w)$, meaning that the difference in effort levels between types is sufficiently small, and a sufficiently large $V_\theta(\theta, w)$, meaning that the absolute cost of effort is sufficiently lower for type $H$. Intuitively, as $w$ increases, the agent’s value $V(\theta, w)$ increases proportionally to the probability of the good outcome.

Second, the principal’s RVE must move in the opposite direction from the agent’s RVE. This means that $S(\theta, w) - W$ must be log-supermodular. Condition 5 implies that $e_{w,\theta}(\theta, w)$ is large, meaning that the reaction of the high-powered incentive ($w$) is different between types, and that $S_\theta(\theta, w)$ and $e_\theta(\theta, w)$ are sufficiently small, meaning that the absolute gain

\(^{24}\)Although $\theta$ is binary, we assume that $H$ and $L$ are real numbers and the functions are well defined for $\theta \in [L, H]$, so that we can take a derivative with respect to $\theta$. 

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from employing a type $H$ agent is not too large. Intuitively, as the wage $w$ increases, the agent’s value $V(\theta, w)$ increases through the probability of the good outcome, while the principal’s value increases directly through the higher effort.

**Characterization of the Optimal Contract under RVE Co-Movement**

We now provide a more detailed characterization of the optimal contract. First, consider the case in which both $V(\theta, w)$ and $S(\theta, w) - W$ are log-submodular. Given Lemma 1 and Proposition 2, it follows that $p_H = 1$ and $v_H = v_L = 0$. Substituting constraint $IC_L$ into (4), we obtain

$$
\max_{w_L \geq w_H \geq 0} \mu_H (\pi(H, w_H) - W) + \mu_L \frac{V(L, w_H)}{V(L, w_L)} (\pi(L, w_L) - W).
$$

(10)

**Proposition 3 (Co-moving MRS)** If both $V(\theta, w)$ and $S(\theta, w) - W$ are log-submodular, then the optimal contract solves (10).

In Section 5, we provide an example in which the optimal contract features $w_H < w_L$ and $p_H > p_L$. This result is markedly different from the results obtained by Gottlieb and Moreira (2014), who study a similar problem, but without an outside option for the principal. In their environment, all agents are retained and offered the same contract. This highlights the difference brought about by the existence of the outside option.\(^{25}\) The fact that the principal has an outside option creates a non-trivial cost of adverse selection – retaining the agent implies giving up the outside option $W$. Hence, the principal can benefit from lowering $p_L$.

**Characterization of the Optimal Contract under RVE Counter-Movement**

Next, consider the case in which $S(\theta, w) - W$ is log-supermodular. Since RVE of the principal and that of the agent counter-move, both $(IC_H)$ and $(IC_L)$ hold with equality. Substituting

\(^{25}\) Although Gottlieb and Moreira (2014) only consider the deterministic contract, we can show that allowing stochastic replacement without outside options does not change the result that all agents are always retained and offered the same contract. Details are available upon request.
Proposition 4 (Counter-Moving MRS) If $V(\theta, w)$ is log-submodular and $S(\theta, w) - W$ is log-supermodular, then the optimal contract solves (11). Moreover, a positive fixed payment is offered ($v_L > 0$) if and only if $w_H > w_L$.

Section 5 contains an example with counter-moving RVE. Again, the comparison with the results in Gottlieb and Moreira (2014) underscores the importance of the firm’s outside option. Lowering $p_H$ is not so costly if the outside option is sufficiently attractive, since the principal can at least obtain the outside option. Without outside option, lowering $p_H$ would be too costly, and the optimal contract would be the same for both types.

4.5 Decoupling and the First Order Approach

One may wonder if it would be possible to solve the problem by the methods previously suggested in the literature: decoupling and first order approach. Since we have the multi-dimensional problem, decoupling suggested by Chade and Swinkels (2016) cannot be applied.

For the first order approach, the difficulty is to control “double deviation,” that is, the case when type $\theta$ reports his type as $\theta'$ and takes an effort level not corresponding to either type $\theta$’s or type $\theta'$’s equilibrium effort. To avoid this problem, we could have assumed additive separability of the effect of types and efforts on outcomes. For example, we could let the probability of the good outcome equal $q(e) + \chi(\theta)$ and the cost of effort equal $c(e)$. In this case, the type does not affect the first order condition for the optimal effort. Given the contract $C$, each type supplies the same effort; however, his additive separability makes both $V(\theta, w)$ and $S(\theta, w) - W$ log-submodular, which is only one of the cases we examine, and the case in which the fixed payment is not used.

26This specification is similar to Garrett and Pavan (2012)
4.6 Comparison to the Social Welfare Maximizing Contract

Finally, we derive the contract that maximizes social welfare, and compare it to the contract that maximizes the principal’s profit. Consider the problem for a social planner who offers employment contracts \((p_\theta, w_\theta, v_\theta)\) in order to maximize

\[
\max \mathbb{E}_\theta [p_\theta \cdot S(\theta, w_\theta) + (1 - p_\theta) \cdot W],
\]

subject to the incentive compatibility constraints for the type \(H\) agent and the type \(L\) agent, respectively:

\[
\begin{align*}
& p_H V (H, w_H) + v_H \geq p_L V (H, w_L) + v_L; \\
& p_L V (L, w_L) + v_L \geq p_H V (L, w_H) + v_H.
\end{align*}
\]

If the social planner retains the agent, she obtains \(S(\theta, w_\theta)\), while if she does not retain the agent, she obtains the exogenous continuation value \(W\).

**Proposition 5** The contract offered by the social planner takes one of two forms:

1. (Retention of type \(H\) only) If \(W > S(L, 1)\), then \(w_H = 1, p_H = 1, v_H = 0,\) and \(p_L = 0, w_L = 0, v_L = V(L, 1)\).
2. (Retention of both agents) If \(W \leq S(L, 1)\), then \(w_\theta = 1, p_\theta = 1, v_\theta = 0\) for \(\theta = H, L\)

**Proof.** In Appendix A.12. ■

Since utility is transferable, the social planner cares only about incentivizing the maximum effort and hiring the right type. For the first goal – that of maximizing effort – we obtain the well-known results that she “sells the project to the worker” in order to generate the maximum effort. For the second goal – that of hiring the right type –, since the exogenous continuation payoff satisfies \(W < S(H, 1)\), the social planner always benefits from keeping the type \(H\) agent; however, keeping the type \(L\) agent is optimal only if the continuation value \(W\) is smaller than the social value of retaining the type \(L\) agent, \(S(L, 1)\).
When comparing this contract to the one that maximizes the principal’s profit, notice that the difference in results comes from the fact that the profit-maximizing principal also cares about leaving enough profit to herself. Hence, the solution is in general different.\(^{27}\)

5 Numerical Illustration

In what follows, we provide numerical examples for both the co-movement and the counter-movement cases. We assume that the good outcome takes value \(y = 10\). This re-scaling allows us to minimize computational errors without qualitatively changing the results.\(^{28}\)

Example Where a Fixed Payment is not Optimal

Consider the case in which \(\mu_H = .7\) and the functions \(q(e)\) and \(c(\theta, e)\) take the following forms: \(q(e) = q_1 \cdot e\), \(c(H, e) = \frac{1}{80} \left( \frac{1}{1-e} - e - 1 \right)\), \(c(L, e) = \frac{1}{40} \left( \frac{1}{1-e} - 1 \right)\).

Under these specifications, \(S(\theta, w) - W\) is log-submodular for each \(W\) with \(S(\theta, w) \geq W\), and \(V(\theta, w)\) is log-submodular. Hence, the RVE of the principal and the agent co-move, and Propositions 1 and 3 characterize the optimal contract.

Figure 2 shows the form of the optimal contract when \(q_1 = .3\) and \(W\) varies between 2.25 and 2.75. As \(q_1\) and \(W\) are very low, both types are inefficient, and the outside option is too low to be worth eliminating any of them. Since both types have similarly low productivity, the principal does not find it optimal to offer separate contracts. As \(W\) increases, the adverse selection problem increases, and offering different contracts becomes optimal. For example, if \(W = 2.45, 2.5, 2.55\), for \(q_1 = .3\), the \(L\) type is very inefficient, and the firm would benefit more if it could access \(W\) instead. The principal would like to set \(p_H > p_L\), which can be achieved in an incentive compatible contract if \(w_H < w_L\). For \(W\) sufficiently large, not hiring any agent and receiving the outside option is optimal.

\(^{27}\)The exception is the case when \(p_L = 0\) or is very close to 0. In the latter case, no matter how high \(w_L\) is, \((IC_H)\) is not binding when she sets \(v_L\) to satisfy \((IC_L)\). Since only \((IC_L)\) binds, the principal maximizes the social welfare and gives a fixed rent to type \(L\). Hence, her incentive is the same as the social planner’s.

\(^{28}\)With \(y = 1\), everything is the same by also rescaling \(c(\theta, e)/10\).
We can also examine the effect of increasing $q_1$. In general, as $q_1$ becomes larger, the threshold $W$ between the same contract being optimal and the different contracts being optimal becomes larger; and the threshold $W$ between the different contracts being optimal and no hire being optimal becomes larger as well.

Finally, we can provide an example where the optimal contract retains only the type $H$ agent and provides a payment for the type $L$ agent to select out. Setting $\mu_H = .99$, $q_1 = .8$, and $W = S(L, 1) + 0.01$ delivers this case. The intuition behind the result is that the probability of encountering a type $L$ is small and $W$ is greater than $S(L, 1)$, so never retaining this type is almost costless for the principal.

**Example where a Fixed Payment is Optimal**

For counter-moving RVE, consider the case in which $\mu_H = .5$, $W = 3.41$ and the functions $q(e)$ and $c(\theta, e)$ take the following forms:

\[
q(e) = .18 + .3e, \quad c(H, e) = .5e + \max\{e - .5, 0\} + .4\max\{e - .8, 0\}, \quad c(L, e) = e + 2.99\max\{e - .5, 0\}.
\]

The cost functions are piecewise linear. It immediately follows that the optimal wages are in the set $\{0, \frac{5}{3}, \frac{10}{3}, \frac{19}{3}, \frac{29.9}{3}\}$, corresponding to the marginal cost divided by $q_1 = .3$. If we

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29 One may notice that the example below does not satisfy (1). We can make the cost function continuously differentiable by modifying it slightly around the kink to satisfy (1). We keep linearity to simplify the algebra.
restrict our attention to those wages, we can show that $S(\theta, w) - W$ is log-supermodular and $V(\theta, w)$ is log-submodular. Intuitively, the cost function ensures that the high type reacts to the incentive much more than the low type, which makes $S(\theta, w) - W$ is log-supermodular. Moreover, $\pi(\theta, w)$ is concave in $w$.\footnote{We use $\epsilon_{w\epsilon\epsilon} \geq 0$ only to show the concavity of $\pi(\theta, w)$ in $w$; hence, piecewise linearity does not change the statement of the lemmas or propositions.} In addition, $S(L, 1) > W = 3.41$. Hence, Proposition 3 characterizes the optimal contract.\footnote{Lemma 5 in Appendix ensures that $(IC_L)$ binds in this problem.}

We can show that, if the principal is forced to offer the same wage, then she offers $w_H = w_L = \frac{10}{3}$ and the expected profit is 6.333. When the principal offers differentiated contracts by setting $w_H = \frac{10}{3}$ and $w_L = \frac{10}{3}$ to induce more effort by type $H$ (solving the moral hazard problem), we can show that the principal’s payoff is increased. By Proposition 3, we have $v_L > 0$ at the optimal contract. Note that $W$ is close to the profit coming from the same wage contract. Hence, the principal does not bare a high cost of reducing $p_H$ in order to solve the moral hazard problem.

Other Functional Forms

The above examples contain functional forms that result in a log-submodular form for $V(\theta, w)$ and a regular form for $S(\theta, w) - W$. This is consistent with the assumptions made for our main analysis, which allow us to present the main results with greater clarity. One may also be interested in the results with other standard functional forms for the cost function, for instance the quadratic form. With quadratic costs in particular, the agent’s payoff $V(\theta, w)$ is log-supermodular. In Appendix F, we perform the complementary analysis for the case in which $V(\theta, w)$ is log-supermodular, and we provide such a numerical example with quadratic costs.
6 Robustness And Extensions

6.1 Outside Option for Agents

In the following extension, we consider the case with exogenous outside options for both the principal and the agent. As before, the firm has outside option $W$. The agent of type $\theta$ has an outside option $V_\theta^O$. We assume that $V_H^O \geq V_L^O$, that is, type $H$ has a higher outside option. In particular, $V_H^O > V_L^O$ implies that the more productive type in the current match is genuinely more productive and so obtains higher value in the outside option. An equality $V_H^O = V_L^O$ implies that type is match-specific.

The principal’s problem is

$$\max_{p_H, w_H, p_L, w_L, v_L} \mu_H [p_H (\pi (H, w_H) - W) - v_H] + \mu_L [p_L (\pi (L, w_L) - W) - v_L],$$

subject to the constraints $p_\theta [V (\theta, w_\theta) - V_\theta^O] + v_\theta \geq \max \{p_\theta [V (\theta, w_\theta) - V_\theta^O] + v_\theta, 0\}$, which capture both the incentive compatibility and the participation constraint for each type $\theta \in \{H, L\}$.

The following is the counterpart of Assumption 2:

**Assumption 4** The following conditions hold: $W > \arg \max_w \pi (L, w)$ and $V (L, w) \geq V_L^O$.

The results of Lemma 2 are readily generalized:\textsuperscript{32}

**Lemma 4** In the optimal contract, the high ability type receives no fixed payment ($v_H = 0$). Moreover, if $p_L > 0$ and $v_L > 0$ then $(IC_H)$ is binding.

**Proof.** Analogous to the proof of Lemma 2. \hfill $\blacksquare$

Since Assumption 4 implies that the principal always wants to decrease $p_L$, if the $L$ type leaves voluntarily due to the participation constraint, it is beneficial to the principal. Hence, for the $L$ type, the incentive constraint is equivalent to $p_L [V (L, w_L) - V_L^O] + v_L = p_H [V (L, w_H) - V_H^O] \geq 0$.

\textsuperscript{32}As before, we focus on the non-trivial contract.
A main difference from the baseline case emerges in the condition of when to replace the $L$ type completely: 

**Proposition 6** If $V^O_H = V^O_L$, then not retaining the type $L$ agent ($p_L = 0$) is optimal if and only if $S(L, 1) - V^O_L - W \leq 0$. If $V^O_H > V^O_L$, then not retaining the type $L$ agent ($p_L = 0$) is optimal only if $S(L, 1) - V^O_L - W \leq 0$.

**Proof.** In Appendix B.1. ■

When the agents have the same outside option, Proposition 6 shows that not retaining type $L$ is equivalent to a negative net social social gain from keeping this type. The intuition is that, if $p_L$ were to be positive, then the principal could increase her objective by decreasing $p_L$ and increasing $v_L$ so as to keep type $L$ indifferent. Moreover, the change would be incentive compatible for type $H$, since this type obtains a higher value from working on the project than type $L$. Similarly, if $S(L, 1) - V^O_L - W > 0$ and $p_L = 0$, then the principal could obtain a higher payoff by increasing $p_L$, offering $w_L = 1$, and reducing $v_L$, so as to keep the rent to type $L$ constant.

With different outside options ($V^O_H > V^O_L$), the problem for the principal changes when $S(L, 1) - V^O_L - W \leq 0$. In this case, the principal cannot reduce $p_L$ if the incentive compatibility constraint for type $H$ is binding. Intuitively, the $H$ type has a higher outside option, and if the $L$ type’s contract allows the $L$ type to capture the outside option with a higher probability – due to the lower probability of retention –, then such a contract may be more attractive to the $H$ type. On the other hand, if $S(L, 1) - V^O_L - W > 0$, increasing $p_L$ and reducing $v_L$ is still a profitable deviation from $p_L = 0$. Therefore, $p_L = 0$ is optimal only if $S(L, 1) - V^O_L - W \leq 0$, but not always. This means that offering a fixed payment $v_L > 0$ in order to completely remove the type $L$ agent becomes harder.

**Proposition 6** highlights the key importance of the nature of the agents’ types, specifically whether the agent type is related to the match-specific fit or to genuine productivity. In our motivating example of the Zappos policy, it was emphasized that the company wants to

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33 See Appendices B.2 and B.3 for the full characterization.
34 This contract would be feasible since $(IC_H)$ is redundant for sufficiently small $p_L$. 

30
encourage “bad fits” to voluntarily leave. If the types that they face are match-specific fits and $V_H^O = V_L^O$, then offering $v_L > 0$ to select out a bad fit is easier. On the other hand, if the types also reflect differences in outside options, then it is more difficult to implement a contract that pays the type $L$ agents to leave. A famous example is provided by Apple. When the company introduced a generous retirement plan to incentivize voluntary early retirement, only those of type $H$, with a higher outside option, left the company and moved to other companies.\footnote{Discussion in Kreps (2003).}

### 6.2 More Than Two Outcomes

We also consider the extension to the case in which more than two outcomes can be observed: $|Y| \geq 3$. Let $Y = \{y_1, ..., y_{|Y|}\} \supseteq y$ be the set of possible outcomes with $y_1 \leq y_2 \leq \cdots \leq y_{|Y|}$. Now the rewards after outcomes is expressed by a vector $w_\theta = (w_\theta(y))_y$. Similarly, the social welfare $S(\theta, w_\theta)$, the principal’s profit $\pi(\theta, w_\theta)$, and the agent’s value $V(\theta, w_\theta)$ have $w_\theta$ as an argument. As before, let $v_\theta$ be the payment when the agent is not retained. The complete analysis of this problem is provided in Appendix C.

First, we assume the analogous condition to Assumption 2:

**Assumption 5** We have $W \geq \max_{w \geq 0} \pi(L, w)$.

Given this assumption, we proceed assuming that $(IC_L)$ binds.

In order to reach specific conditions for when $v_L > 0$, we derive a condition analogous to the log-submodularity/supermodularity of $S(\theta, w) - W$ and $V(\theta, w)$. In particular, let us define $S_{|Y|}(\theta, w)$ and $V_{|Y|}(\theta, w)$ as the social welfare and the agent’s payoff given type $\theta$, respectively, when the agent is compensated only after the highest outcome: $w = (0, \ldots, 0, w)$. We want to analyze when $v_L > 0$ based on the log-submodularity/supermodularity of $S_{|Y|}(\theta, w)$ and $V_{|Y|}(\theta, w)$.

The next condition guarantees that type $H$ is paid only after the highest outcome:

**Assumption 6** The following two conditions are satisfied:
1. (Monotone Hazard Rate Condition) The hazard rate \( \frac{q(y_n|e)}{q(y_n|e')} \) is increasing in \( n \).

2. (Monotone Likelihood Ratio Condition) For each \( e \) and \( e' \) with \( e > e' \), \( \frac{q(y_n|e)}{q(y_n|e')} \) is increasing in \( n \).

The first part of the condition corresponds to the monotone hazard rate condition. Without adverse selection, this condition alone would be sufficient to show that, in the optimal contract, the agent is rewarded only after the highest outcome \( y_{Y_1} \). The second part of the above condition implies that rewarding the agent after outcome \( y_{Y_1} \) decreases type \( L \)'s incentive to mimic type \( H \). This implies that the \( H \) type is compensated only after the highest outcome: \( w_H = (0, \ldots, 0, w_H) \). Hence, we can express the social welfare and the agent’s payoff by \( S_{Y_1}(H, w) \) and \( V_{Y_1}(H, w) \) if he is of type \( H \).

For the \( L \) type agent, it is not always the case he is compensated only after the highest outcome. To see why, the monotone hazard rate condition implies that compensating the \( L \) type only after the highest outcome incentivizes more effort – it helps solve the moral hazard problem; however, the monotone likelihood ratio condition implies that compensating the \( L \) type only after the highest outcome also increases the \( H \) type’s incentive to mimic the \( L \) type. Thus, compensating the \( L \) type after intermediate outcomes helps solve the adverse selection problem.\(^{36}\)

The above implies that we cannot always express the social welfare and the agent’s payoff by \( S_{Y_1}(L, w) \) and \( V_{Y_1}(L, w) \) for the \( L \) type agent. Nonetheless, we can derive a condition with which we can relate \( S_{Y_1}(L, w) \) and \( V_{Y_1}(L, w) \) to when the fixed payment is positive.

Specifically, given \( w_L \), let \( w(L, w_L) \) be the reward after \( y_{Y_1} \) such that type \( L \) is indifferent between \( w(L, w_L) = (0, \ldots, 0, w(L, w_L)) \) and \( w_L: V(\theta, w(L, w_L)) = V(\theta, w(L, w_L)) = V(L, w_L) \). As mentioned, this increases the effort of the \( L \) type. Let \( \Delta S(L, w_L) \) be the

\(^{36}\)Note that the argument that the \( H \) type is compensated only after the highest outcome is analogous to Lemma 1 – the \( H \) type is compensated only after \( y = 1 \). The argument that the \( L \) type may be compensated after the intermediate outcomes is analogous to \( v_L > 0 \) in the binary case – \( v_L > 0 \) is costly for the principal, but reducing \( v_L \) and increasing \( w_L \) may violate type \( H \)'s incentive compatibility.
increase of the social welfare when the agent is of type $L$:

$$\Delta S(L, w_L) = S(L, w(L, w_L)) - S(L, w_L) \geq 0.$$ 

At the same time, the $H$ type’s value from $C_L$ also increases:

$$\Delta V(H, w_L) = V(H, w(L, w_L)) - V(H, w_L) \geq 0.$$ 

If the benefit of increasing $S(L, w_L)$ dominates the cost of increasing $\Delta V(H, w_L)$, such that

$$\Delta S(L, w_L) \geq \Delta V(H, w_L) \text{ for each } w_L,$$

then we can show that $v_L \geq 0$ implies the log-supermodularity of $S_{Y|}(\theta, w)$.

**Proposition 7** Suppose Assumptions 6 and (12) are satisfied and $p_L > 0$. In addition, suppose $V_{Y|}(\theta, w)$ is log-submodular and $S_{Y|}(\theta, w) - W$ is regular. Then the optimal contract specifies a fixed payment to the low ability agent ($v_L \geq 0$) only if $S_{Y|}(\theta, w) - W$ is log-supermodular.

**Proof.** In Section C.3. ■

Intuitively, (12) implies that, when the principal deviates and offers the same contract to both types as seen in (8) and (9), she would like to offer the contract which compensates the agent only after the highest outcome. This is sufficient to derive the analogous condition in Proposition 2.

### 6.3 Endogenous Outside Option $W$

Thus far, we presented the characterization of the optimal employment contract when the principal had the exogenous outside option $W$. In what follows, we endogenize $W$ by assuming that, if the current agent is not retained, the principal then pays a small cost $\kappa > 0$.
and draws another agent.\textsuperscript{37} The game then repeats itself until an agent is retained to work on the project.

With the endogenous outside option $W$, the planning problem for the principal becomes

$$
\tilde{J}(W) = \max_{\{p_H,p_L,w_H,w_L,v_H,v_L\}} \mu_H \left[ p_H \pi(H,w_H) + (1 - p_H)(W - \kappa) \right]
+ \mu_L \left[ p_L \pi(L,w_L) + (1 - p_L)(W - \kappa) \right],
$$

subject to

$$
p_H V(H,w_H) + v_H \geq p_L V(H,w_L) + v_L; \quad (IC^E_H)
$$
$$
p_L V(L,w_L) + v_L \geq p_H V(L,w_H) + v_H. \quad (IC^E_L)
$$

In equilibrium, $\tilde{J}(W) = W$ by recursion. Let $\tilde{J}$ be the value of $\tilde{J}(W)$ such that $\tilde{J}(W) = W$.

With the exogenous outside option, Assumption 2 guarantees that $(IC_L)$ binds. With the endogenous outside option, the small cost $\kappa$ guarantees that $(IC_L)$ binds:

\textbf{Lemma 5} If $\pi(L,w_L) < W - \kappa$, then constraint $(IC_L)$ binds. In addition, for a sufficiently small $\kappa$, constraint $(IC_L)$ binds at the fixed point $W = \tilde{J}(W)$.

\textbf{Proof.} In Section D.1. \hfill \square

Constraint $(IC_L)$ binds if $\pi(L,w_L) < W - \kappa$, meaning that the profit the principal is obtaining from hiring the $L$ type is lower than the value of getting a new agent after paying the waiting cost. This happens in equilibrium when the waiting cost $\kappa$ is sufficiently small. To see why, note that the $H$ type is more efficient than the $L$ type as long as $c_{\theta e} < 0$. If the cost of re-sampling is low and constraint $(IC_L)$ were not binding, then the principal would prefer to decrease $p_L$ and increase the probability of a new draw, as the next agent could be an $H$ type.

Given this lemma, we focus on the case in which $\kappa$ is small and constraint $(IC_L)$ binds. Then, the analogues of Propositions 1 and 2 follow:

\textsuperscript{37}We formulate the problem such that a new agent arrives right away, and the principal incurs the direct cost $\kappa$. We can also formalize $\kappa$ by discounting and search friction. Details are available upon request.
Proposition 8 For sufficiently small $\kappa > 0$, the optimal contract can take one of the following forms:

1. The $L$ type is not retained: $p_L = 0$, $p_H = 1$, $v_L > 0$.

2. Both types are hired with positive probability. Moreover, suppose $V(\theta, w)$ is log-submodular and $S(\theta, w) - W$ is regular. Then,

   (a) (RVE Co-Movement) If $S(\theta, w) - W$ is log-submodular, then $v_L = 0$, $1 = p_H \geq p_L$ and $w_L \geq w_H$.

   (b) (RVE Counter-Movement) If $S(\theta, w) - W$ is log-supermodular, then $v_L \geq 0$, and $1 = p_L \geq p_H$ and $w_H \geq w_L$.

Proof. See Appendix D.2.

6.4 Endogenous Outside Option $W$ and Multiple Agent Types

Here we extend the model to consider the case in which the agent can have more than two types. Specifically, we assume that the agent can have types $\theta_1, \ldots, \theta_n$ with $\theta_1 < \theta_2 < \cdots < \theta_{n-1} < \theta_n$ and $V(\theta, w)$ is well defined for $\theta \in \Theta = [\theta, \bar{\theta}] \supset \{\theta_1, \ldots, \theta_n\}$. The cost $c(\theta, e)$ and the marginal cost of effort $c_e(\theta, e)$ are both decreasing in $\theta$. The contract $C_i$ is designed for type $\theta_i$. Let $IC_{i,j}$ be the constraint that type $i$ prefers their contract to claiming to be type $j$:

$$p_i V(\theta_i, w_i) + v_i \geq p_j V(\theta_i, w_j) + v_j.$$ 

As in the previous section, we assume an endogenous outside option $W$ with a small cost of finding a new agent.\textsuperscript{38}

The difficulty with multiple types is in determining whether the payments $v_\theta$, the rewards $w_\theta$, and probabilities $p_\theta$ are following a monotone pattern. The complication is driven by the multi-dimensionality of the problem, which creates a non-trivial preference ordering over

\textsuperscript{38}We do this to avoid a complication arising from non-binding incentive compatibility of lower types.
contracts for different types;\footnote{We can create an example in which, even with globally log-supermodular $S(\theta, w) - W$ and log-submodular $V(\theta, w)$, the base payment $v_0$ of the optimal contract is not monotone in $\theta$. Details are available upon request.} yet, if the variation in the agent’s type $\theta$ is sufficiently large, we can still show that it is optimal to use the fixed payment for lower types:

**Proposition 9** There exist $\bar{\kappa} \in (0, 1)$, $\bar{q} \in (0, 1)$, $\bar{c} \in (0, 1)$, and $\bar{c} \in (0, 1)$ such that if (i) waiting is not costly, $\kappa < \bar{\kappa}$, (ii) effort is sufficiently important, $q_0 < \bar{q}$, (iii) the lowest type is sufficiently unproductive, $c_e(\theta_1, 0) > \bar{c}$, and (iv) the highest type is sufficiently productive, $c_e(\theta_n, \frac{1}{q_1}) < \bar{c}$, then the optimal contract specifies a fixed payment for the lowest type ($v_1 > 0$).

**Proof.** In Appendix E. □

The intuition is as follows. If the highest type is sufficiently productive, then the endogenous outside option $W$ is sufficiently high. Thus, if waiting is not costly and the lowest type is not productive, then it is optimal to replace the low type and open up the possibility of a drawing a more productive agent.

### 7 Empirical and Policy Implications

The model produces testable empirical implications, and it provides a framework to inform the regulatory treatment of Pay to Quit or similar programs. It provides the conditions necessary for Pay to Quit incentives to be used effectively in employee selection. These conditions could be used, for example, to study empirically whether compensation schemes with Pay to Quit or similar incentives are effective means for employee selection or another form of rent-seeking.\footnote{See Bebchuk and Fried (2003) for a discussion of this issue in the context of executive compensation.} This distinction is critical when considering regulation that to either limit or encourage these practices.

First, the model highlights the case in which the firm-specific fit between employer and employee is of crucial importance to the firm, so much so that hiring a wrong fit offers a lower social welfare from the relationship than the firm’s outside option. In this case, paying the
low type to select them out is optimal, and the firms retains only the high ability employee—
“the right fit”. To achieve this outcome, the firm must offer the low ability employee the
option of a payment to quit, so that this type prefers selecting out. This case is illustrated
by the policies implemented by Zappos and Amazon, described in the Introduction.

Second, the model shows that both types of workers may be retained with positive prob-
ability, but not with certainty. In this case, a Pay to Quit incentive is not optimal if there is
co-movement between the firm’s and the employee’s relative values of employment (RVE).
Observing Pay to Quit incentives in these settings would suggest a deviation for optimal
compensation. Moreover, the case of both RVE decreasing in employee type corresponds to
an environment in which there is moderate difference in effort provision by different employee
types – the value $e_\theta (\theta, w)$ is sufficiently large—, and both types of employees respond simi-
larly to monetary incentives – the value $e_{w\theta} (\theta, w)$ is small. This is the case in which the firm
values retaining the “right fits” sufficiently highly – relatively more than incentivizing e-
fort through compensation. Therefore, one would expect to not observe Pay to Quit in industries
where team work is essential for the project success – so that fit matters –, and employees
are not strongly incentivized by monetary rewards – so that there is a moderate difference
in effort provision by types, but monetary compensation does not produce substantial vari-
ation in responses across types. For instance, such a description may apply to emergency
intervention teams, fire fighters or other special intervention units. It may also apply in the
case of teachers, where both compensation and firing have been matters of intense public
policy debates. For the case of teachers in particular, Hanushek et al. (2004) have shown
that monetary compensation does not play a significant role in teachers’ workplace selection.

The model also shows when Pay to Quit incentives should be observed due to optimal
contracting. The specific condition is that of counter-movement of RVE between firm and
employees – one must be decreasing while the other is increasing. This is the case in which
there is substantial variation in the responses of employees to monetary incentives – the
value $e_{w\theta} (\theta, w)$ is large – and yet effort provision does not vary too much between types –
the value $e_\theta (\theta, w)$ is sufficiently small. This may correspond to industries in which generating
high effort through monetary compensation is important for the firm, but the high ability employees value retention relatively more than compensation, compared to the low ability types. Such a description could apply, for example, to law firms or financial management organizations, where incentivizing high effort is very important for the firm, but retention at a prestigious firm may be more relatively more valuable for the worker. In such cases, the Pay to Quit incentive may come in the form of a payment to leave or in the form of intra-firm selection, by placing employees on different tracks within the firm. This may explain, for example, the introduction in recent years of early tracking at corporate law firms, with a partner track as well as a “permanent associate” track, that offers a higher probability of retention, but lower compensation.  

8 Conclusion

We presented a model to study a fundamental problem that arises in employment contracts. We modeled an environment in which a firm offers contracts to employees who have unobservable ability (there is adverse selection), exert unobservable effort (there is moral hazard), and the firm can exercise an outside option only if the current employee is not retained. We characterized the optimal employment contract in this environment and showed that it can take the form of differentiated contracts for different candidate abilities. Moreover, we showed when the optimal contract features a fixed payment, independent of project outcome. This occurs whenever the firm’s and the agent’s marginal rates of substitution of compensation to retention probability change in opposite directions as the agent’s ability increases. The model may be applied to both private sector and public sector employment contracts. It helps inform what features of organizations and what worker characteristics should be examined in order to account for the desirability of Pay to Quit incentives.

References


A Proofs of Results from Section 4

A.1 Concavity/Convexity

In the proofs, the following concavity/convexity results will be useful:

\[
\begin{align*}
e_{\theta w}(\theta, w) &= -q_1 \frac{c_{eel}(\theta, e(\theta, w))}{[c_{ee}(\theta, e(\theta, w))]^2} + \frac{c_{cee}(\theta, e(\theta, w)) c_{ee}(\theta, e(\theta, w))}{[c_{ee}(\theta, e(\theta, w))]^3} \geq 0 \text{ by (1)}; \\
e_{ww}(\theta, w) &= -q_1 e(\theta, w) \frac{c_{ee}(\theta, e(\theta, w))}{[c_{ee}(\theta, e(\theta, w))]^2} \leq 0 \text{ by (1)}; \\
S_w(\theta, w) &= q_1 (1 - w) e_w(\theta, w) \geq 0; \\
S_{w\theta}(\theta, w) &= q_1 (1 - w) e_{\theta w}(\theta, w) \geq 0; \\
S_{ww}(\theta, w) &= -q_1 (e(\theta, w)) e_w(\theta, w) + q_1 e(\theta, w) (1 - w) e_{ww}(\theta, w) \leq 0; \\
V_w(\theta, w) &= q_0 + q_1 e(\theta, w); \\
V_{w\theta}(\theta, w) &= q_1 e_{\theta}(\theta, w) = \frac{c_{ee}(\theta, e(\theta, w))}{c_{ee}(\theta, e(\theta, w))} \geq 0; \\
V_{ww}(\theta, w) &= q_1 e_w(\theta, w) \geq 0; \\
\pi_{ww}(\theta, w) &= S_{ww}(\theta, w) - V_{ww}(\theta, w) \leq 0. \tag{14}
\end{align*}
\]

A.2 The Optimal Contract under Adverse Selection Only

Suppose that type $\theta \in \{H, L\}$ has the probability $q_\theta$ of achieving outcome $y = 1$. This probability is independent of effort and $q_H > q_L$. The rest of the setup is as described above.

First, we notice that, if the agent has no outside option, then the principal’s problem is to choose $\{p_{\theta}, w_{\theta}\}_{\theta \in \{L, H\}}$ to solve the following program:

\[
\max p_H \cdot (q_H \cdot (1 - w_H) - W) - v_H + p_L \cdot (q_L \cdot (1 - w_L) - W) - v_L
\]

subject to

\[
\begin{align*}
p_{\theta} \cdot q_H \cdot w_H + v_H &\geq p_L \cdot q_H \cdot w_L + v_L; \tag{15} \\
p_{\theta} \cdot q_L \cdot w_H + v_H &\leq p_L \cdot q_L \cdot w_L + v_L. \tag{16}
\end{align*}
\]
The constraints represent the incentive compatibility constraints for types $H$ and $L$, respectively. In this problem, it follows immediately that $p_H = 1$ and $p_L = w_H = v_H = v_L = 0$ yields the highest payoff to the principal. The optimal contract for the principal is to retain the $H$ type agent and make no payments to the agent.

To derive more insight from this problem, we therefore need to assume that the agent has an outside option. Denote the outside option of type $\theta$ by $V_\theta$. The principal’s maximization problem thus becomes

$$\max_{\{p_0, w_0\}_{\theta \in \{L, H\}}} p_H \cdot [q_H \cdot (1 - w_H(y = 1)) - (1 - q_H) \cdot w_H(y = 0) - W] - v_H$$

$$+ p_L \cdot [q_L \cdot (1 - w_L(y = 1)) - (1 - q_L) \cdot w_L(y = 0) - W] - v_L$$

subject to

$$p_H \cdot [q_H w_H(y = 1) + (1 - q_H) w_H(y = 0)] + (1 - p_H) \cdot V_H + v_H \geq$$

$$p_L \cdot [q_H w_H(y = 1) + (1 - q_H) w_H(y = 0)] + (1 - p_L) \cdot V_H + v_L$$

(17)

$$p_L \cdot [q_L w_L(y = 1) + (1 - q_L) w_L(y = 0)] + (1 - p_L) \cdot V_L + v_L \geq$$

$$p_H \cdot [q_L w_L(y = 1) + (1 - q_L) w_L(y = 0)] + (1 - p_H) \cdot V_L + v_H$$

(18)

$$p_H \cdot [q_H w_H(y = 1) + (1 - q_H) w_H(y = 0)] + (1 - p_H) \cdot V_H + v_H \geq V_H$$

(19)

$$p_L \cdot [q_L w_L(y = 1) + (1 - q_L) w_L(y = 0)] + (1 - p_L) \cdot V_L + v_L \geq V_L.$$  

(20)

The first two inequalities represent the incentive compatibility constraints for types $H$ and $L$, respectively, and the last two inequalities are the participation constraints for each type.

In this problem, if $q_H V_L \geq q_L V_H$, then it is optimal to set $v_H = 0$, $p_H = 1$, $q_H w_H = V_H$, and $p_L = w_L = v_L = 0$, and have the $L$ type take the outside option. Hence, we focus in what follows on the non-trivial case of $q_L V_H > p_H V_L \geq 0$, and in which $p_H > 0$ in the optimal contract.

The principal’s problem leads to the following implications for the compensation agents. For the agent of type $H$, it is preferable for the principal to pay a wage after an outcome
$y = 1$ and to pay no wage after $y = 0$. Compensating the $H$ type through $w_H(y = 1)$ costs the principal fewer resources, and it discourages the $L$ type from claiming to be an $H$ type. Similarly, the principal does better by paying type $L$ after the outcome $y = 0$, and not after outcome $y = 1$. By paying only after $y = 0$ rather than after $y = 1$, the principal can spend less and still give the $L$ type agent the same expected value; moreover, this strategy also reduces the benefit for the $H$ type from claiming to be type $L$. This compensation scheme dominates the use of a payment to quit whenever type $L$ is retained with positive probability, so $p_L > 0$. The only instance when a payment to quit may be used is if the principal wished to always remove the type $L$ agent. Therefore, we obtain the following result.

**Proposition 10** Under adverse selection only, with the outside options for the agents satisfying $q_L V_H > p_H V_L \geq 0$, the optimal contract has the following properties:

- Either $p_L > 0$, in which case no payment to quit is offered ($v_H = v_L = 0$) and $w_H(y = 1) > 0$, $w_L(y = 0) > 0$ and $w_H(y = 0) = w_L(y = 1) = 0$, or

- Type $L$ is not retained ($p_L = 0$) and $v_L > 0$, $w_H(y = 1) > 0$ and $w_H(y = 0) = v_H = 0$. This case happens only if $q_L - W < 0$.

**Proof.** Denote by $w_y^H$ the payment after outcome $y$. We focus on the case in which $p_H > 0$ in the optimal contract.

To type $H$, it is better to pay by $w_H^1$ rather than $w_H^0$: To increase the value to the high type by $\Delta$, we need to increase $w_H^1$ by $\frac{\Delta}{p_H q_H}$. This costs the principal $\Delta$; and the low type’s value when he pretends to be the high type increases by $\frac{q_L \Delta}{q_H} < \Delta$. On the contrary, if the principal increases $w_H^0$ or $v_H$, then it costs the principal $\Delta$ and the low type’s value when he pretends to be the high type increases by $\frac{1-q_L}{1-q_H} \Delta > \Delta$ (in case of increasing $w_H^0$) or $\Delta$ (in case of increasing $v_H$).

Similarly, to type $L$, it is better to pay by $w_L^0$ rather than $w_L^1$ if $p_L > 0$: To increase the value to the low type by $\Delta$, we need to increase $w_L^1$ by $\frac{\Delta}{p_L (1-q_L)}$. This costs the principal $\Delta$; and the high type’s value when he pretends to be the low type increases by $\frac{1-q_H}{1-q_L} \Delta < \Delta$. 

43
On the contrary, if the principal increases \( w^1_L \) or \( v_L \), then it costs the principal \( \Delta \) and the high type’s value when he pretends to be the low type increases by \( \frac{q_H}{q_L} \Delta > \Delta \) (in case of increasing \( w^1_L \)) or \( \Delta \) (in case of increasing \( v_L \)).

Hence, if \( p_L > 0 \) is optimal, then the problem becomes

\[
\max p_H \left( q_H \left( 1 - w^1_H \right) - W \right)
\]

subject to

\[
\begin{align*}
p_H \left( q_H w^1_H - V_H \right) & \geq \max \left\{ 0, p_L \left( (1-q_H) w^0_L - V_H \right) \right\} \quad (IC_H) \\
p_L \left( (1-q_L) w^0_L - V_L \right) & \geq \max \left\{ 0, p_H \left( q_L w^1_H - V_L \right) \right\} \quad (IC_L)
\end{align*}
\]

Therefore, it is weakly dominant strategy for the principal not to use the fixed payment. Intuition is simple: given \( y = 1 \), the agent is more likely to be a high type. Hence, use \( w^1_H \) to compensate the high type; given \( y = 0 \), the agent is more likely to be a low type. Hence, use \( w^0_L \) to compensate the low type.

Note that this simple intuition breaks down if there is moral hazard: using \( w^0_L \) disincentivizes agents from putting any effort and if the effort is sufficiently important, then the principal does not want to compensate the low type only after \( y = 0 \). Hence, it is important to consider the agent’s incentive seriously.

Next, if \( p_L = 0 \) is optimal, then the principal’s payoff is

\[
\max p_H \left( q_H \left( 1 - w^1_H \right) - W \right) - v_L
\]

subject to

\[
\begin{align*}
p_H \left( q_H w^1_H \right) + (1 - p_H) V_H & \geq \max \left\{ V_H, V_H + v_L \right\} \quad (IC_H) \\
v_L + v_L & \geq \max \left\{ V_L, p_H \left( q_L w^1_H \right)+ (1 - p_H) V_L \right\} \quad (IC_L)
\end{align*}
\]
\[ p_H \left( q_H w^1_H - V_H \right) \geq v_L \quad \text{((IC_H))} \]
\[ v_L \geq p_H \left( q_L w^1_H - V_L \right) \quad \text{((IC_L))} \]

Hence, \((IC_H)\) and \((IC_L)\) imply \((q_H - q_L) w^1_H \geq V_H - V_L\). If this is not binding, then it is optimal to set \( v_H = 0, \ p_H = 1, \ q_H w_H = V_H, \) and \( p_L = w_L = v_L = 0 \). Hence, we have 
\[ w^1_H = \frac{V_H - V_L}{q_H - q_L} \] and 
\[ v_L = p_H \left( q_H \left( \frac{V_H - V_L}{q_H - q_L} - V_H \right) \right) = p_H \left( \frac{q_L V_H - q_H V_L}{q_H - q_L} \right). \]

Since the problem now becomes proportional to \( p_H \), we have \( p_H = 1, \ w^1_H = \frac{V_H - V_L}{q_H - q_L}, \) and 
\[ v_L = \frac{q_L V_H - q_H V_L}{q_H - q_L}. \]

Now, let us verify the optimality of this strategy. Suppose the principal reduces \( v_L \) by \( \Delta \) and increase \( p_L \) by \( \Delta_p \). As mentioned, it is optimal to compensate the low type after the bad outcome. To keep the low type indifferent, she will increase \( w^0_L \) by \( \frac{\Delta}{\Delta_p} \). Since the high type receives less payoff than the low type from \( w^0_L \), this change does not violate \((IC_H)\). This change brings the extra payoff of \( \Delta_p (q_L - W) \) to the principal. Hence, \( v_L > 0 \) is optimal only if \( q_L - W < 0 \). \( \blacksquare \)

A.3 Proof of Lemma 1

A.3.1 Proof that \( v_H = 0 \)

Assume \( v_H > 0 \). Note that \((IC_H)\) is binding since otherwise the principal would reduce \( v_H \):

\[ V(H, w_H) + v_H = V(H, w_L) + v_L. \] (21)
From (4), the principal’s payoff from the type $H$ is

$$p_H \left( S(H, w_H) - V(H, w_H) \right) + (1 - p_H) W - v_H$$

$$= p_H \left( S(H, w_H) - W \right) + W - p_L V(H, w_L) - v_L.$$

Therefore, offering $v_H - \Delta_v$ and $w_H + \Delta_w$ to keep (21) improves the principal’s payoff by (14). Moreover, $(IC_L)$ is satisfied:

$$V(L, w_H + \Delta_w) + v_H - \Delta_v$$

$$= V(L, w_H) + v_H + V(L, w_H + \Delta_w) - V(L, w_H) - \Delta_v$$

$$\leq V(L, w_L) + v_L + V(L, w_H + \Delta_w) - V(L, w_H) - \Delta_v$$

by $(IC_L)$ for the original contract

$$\leq V(L, w_L) + v_L + V(H, w_H + \Delta_w) - V(H, w_H) - \Delta_v.$$

The last line follows since $V_{\theta w}(\theta, w) \geq 0$ by (14). Since we keep (21), this inequality implies

$$V(L, w_H + \Delta_w) + v_H - \Delta_v \leq V(L, w_L) + v_L.$$

Therefore, we have $v_H = 0$.

**A.3.2 Proof that either $p_L = 1$ or $p_H = 1$**

Assume next $p_H < 1$ and $p_L < 1$. From (4), the principal’s objective is

$$\mu_H \left[ p_H \pi(H, w_H) + (1 - p_H) W - v_H \right] + \mu_L \left[ p_L \pi(L, w_L) + (1 - p_L) W - v_L \right]$$

$$= \mu_H \left[ p_H \left[ \pi(H, w_H) - W \right] - v_H \right] + \mu_L \left[ p_L \left[ \pi(L, w_L) - W \right] - v_L \right] + W.$$

We have

$$\mu_H \left[ p_H \left[ \pi(H, w_H) - W \right] - v_H \right] + \mu_L \left[ p_L \left[ \pi(L, w_L) - W \right] - v_L \right] > 0$$
since otherwise \( p_H = p_L = v_H = v_L = 0 \) would be optimal (recall that we proceed with the guess that \( \max \{ p_H, p_L \} > 0 \)).

Offering \((kp_H, w_H, kv_H)\) and \((kp_L, w_L, kv_L)\) with \( k > 1 \) increases the principal’s profit:

\[
\mu_H [kp_H \pi (H, w_H) + (1 - kp_H) W - kv_H] + \mu_L [kp_L \pi (L, w_L) + (1 - kp_L) W - v_L]
= k \{ \mu_H [p_H [\pi (H, w_H) - W] - v_H] + \mu_L [p_L [\pi (L, w_L) - W] - v_L] \} + W.
\]

Since \((IC_H)\) and \((IC_L)\) are satisfied, this is a profitable deviation, which is a contradiction.

**A.4 Proof of Lemma 2**

If \( v_L > 0 \) and \((IC_L)\) is not binding, then the principal would simply reduce \( v_L \) (regardless of Assumption 2). Suppose \( v_L > 0 \) and \((IC_H)\) is not binding. Then, reducing \( v_L > 0 \) and increasing \( w_L \) to keep \( p_L V (L, w_L) + v_L \) improves the principal’s payoff and satisfies the incentive compatibility constraint. The proof is the same as Lemma 1.

**A.5 Proof of Lemma 3**

If \( v_L = 0 \), then \((IC_H)\) implies \( p_H \geq p_L \) if \( w_H \leq w_L \), and \((IC_L)\) implies \( p_L \geq p_H \) if \( w_H \geq w_L \).

If \( v_L > 0 \), then both \((IC_H)\) and \((IC_L)\) bind from Lemma 2. Hence

\[
p_H V (H, w_H) = p_L V (H, w_L) + v_L;
p_H V (L, w_H) = p_L V (L, w_L) + v_L,
\]

and so

\[
\frac{p_H}{p_L} = \frac{V (H, w_L) - V (L, w_L)}{V (H, w_H) - V (L, w_H)}.
\]

Since \( V_{\theta w} (\theta, w) = q_1 e_\theta (\theta, w) > 0 \) by (14), we have \( p_H \geq p_L \) if \( w_H \leq w_L \), and \( p_L \geq p_H \) if \( w_H \geq w_L \). (Use (5) in case \( V \) is not differentiable.)
A.6 Proof of Proposition 1

Given Lemma 1, it suffices to prove that $p_L = 0$ is optimal if and only if $S(L, 1) - W \leq 0$. Since $(IC_L)$ binds, it follows that $v_L = p_H V(L, w_H) - p_L V(L, w_L)$. Substituting $v_L$ into the principal’s problem,

$$
\max_{p_H, p_L, w_H, w_L} \mu_H p_H [q(e(H, w_H))(1 - w_H) - W] \\
+ \mu_L [p_L q(e(L, w_L))(1 - w_L) - p_L W - v_L]
$$

$\Leftrightarrow$

$$
\max_{p_H, w_H} \mu_H p_H [q(e(H, w_H))(1 - w_H) - W] - p_H \mu_L V(L, w_H) \\
+ \mu_L \max_{p_L, w_L} [p_L q(e(L, w_L)) - c(L, e(L, w_L)) - W]
$$

subject to $(IC_H)$.

If and only if $\max_{w_L} [q(e(L, w_L)) - c(L, e(L, w_L)) - W] \leq 0$, the optimal $p_L$ is equal to 0 since, at $p_L = 0$, $(IC_H)$ is slack. Moreover, $q(e(L, w_L)) - c(L, e(L, w_L))$ is the social welfare from the low type, so

$$
\max_{w_L} [q(e(L, w_L)) - c(L, e(L, w_L)) - W] = S(L, 1) - W,
$$
as desired.

A.7 Useful Lemma Given Binding $(IC_L)$

The following lemma pins down the shape of the optimal contract given log-submodularity (or log-supermodularity) of $V$:

**Lemma 6** If $(IC_L)$ holds with equality, then,

1. If $V$ is log-submodular, then $w_H > w_L$ and $p_H < p_L$ if $v_L > 0$; and $w_H \leq w_L$ and $p_H \geq p_L$ if $v_L = 0$. 

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2. If $V$ is log-supermodular, then $w_H < w_L$ and $p_H > p_L$ if $v_L > 0$; and $w_H \geq w_L$ and $p_H \leq p_L$ if $v_L = 0$.

We now prove Lemma 6, focusing on log-submodular $V$, since the argument for log-supermodular $V$ is symmetric.

A.7.1 Proof: If $v_L > 0$, then $w_H > w_L$ and $p_H < p_L$

Since Lemma 2 implies both ($IC_L$) and ($IC_H$) are binding, rearranging them yields

\[
\left\{ \begin{array}{l}
  p_H = p_L \frac{V(H, w_L) - V(L, w_L)}{V(H, w_H) - V(L, w_H)}; \\
p_H V(L, w_H) = p_L V(L, w_L) + v_L.
\end{array} \right. \tag{22}
\]

Since $v_L > 0$, we have $p_H V(L, w_H) > p_L V(L, w_L)$, and so

\[
\frac{V(H, w_L) - V(L, w_L)}{V(H, w_H) - V(L, w_H)} V(L, w_H) > V(L, w_L) \iff V(H, w_L) V(L, w_H) > V(H, w_H) V(L, w_L).
\]

Hence we have $w_H > w_L$ if $V$ is log-submodular. (22) implies $p_H < p_L$.

A.7.2 Proof: If $v_L = 0$, then $w_H \leq w_L$ and $p_L \geq p_H$

Since ($IC_L$) is satisfied with equality, it follows that $p_H V(L, w_H) = p_L V(L, w_L)$. If $V$ is log-submodular and $w_H > w_L$, then $V(H, w_H) / V(L, w_H) < V(H, w_L) / V(L, w_L)$, and so $p_H V(H, w_H) < p_L V(H, w_L)$. This contradicts to ($IC_H$). Hence $w_H \leq w_L$, and so $p_H / p_L = V(L, w_L) / V(L, w_H) \geq 1$.

A.8 Proof of Proposition 2

If she offers $C_H$ to type $L$, the principal’s profit changes by

\[
0 \geq [p_H \pi(L, w_H) + (1-p_H)W] - [p_L \pi(L, w_L) - v_L + (1-p_L)W] \\
\geq p_H [S(L, w_H) - W] - p_L [S(L, w_L) - W] \text{ by (IC}_L\text{).} \tag{23}
\]
On the other hand, if she offers $C_L$ to type $H$, her profit changes by

$$0 \geq [p_L \pi (H, w_L) - v_L + (1 - p_L) W] - [p_H \pi (H, w_H) + (1 - p_H) W]$$
$$\geq p_L [S (H, w_L) - W] - p_H [S (H, w_H) - W] \text{ by (IC}_H).$$

Rearranging, we obtain

$$p_H [S (H, w_H) - S (L, w_H)] \geq p_L [S (H, w_L) - S (L, w_L)].$$

Since (23) implies $p_H \leq p_L [S (L, w_L) - W] / [S (L, w_H) - W]$, we obtain

$$S (L, w_L) - W \geq S (L, w_H) - W \quad [(S (H, w_H) - W) - (S (L, w_H) - W)]$$
$$\geq [(S (H, w_L) - W) - (S (L, w_L) - W)]$$

$$\Leftrightarrow (S (L, w_L) - W) (S (H, w_H) - W) \geq (S (L, w_H) - W) (S (H, w_L) - W).$$

Hence if the net social welfare is log-supermodular (or log-submodular), then $w_H \geq w_L$ (or $w_H \leq w_L$). Given Lemma 6 and $V (\theta, w)$ log-submodular, it follows that $v_L > 0$ if and only if $w_H > w_L$.

**A.9 Proof of Corollary 1**

Log-submodularity (or supermodularity) of $F$ is equivalent to

$$\frac{d^2 \log F (\theta, w)}{dwd\theta} \leq (\geq) 0.$$

By the envelope theorem, the sign of $\frac{d^2 \log V (\theta, w)}{dwd\theta}$ is determined by

$$q_1 e_\theta (\theta, w) V (\theta, w) - [q_0 + q_1 e (\theta, w)] V_\theta (\theta, w)$$
and that of $\frac{d^2 \log(S(\theta,w)-W)}{dw d\theta}$ is determined by

$$q_1 (1 - w) e_\theta (\theta, w) [S(\theta, w) - W] - q_1 (1 - w) e_\theta (\theta, w) S_\theta (\theta, w).$$

The conclusion about the sufficient magnitudes of $e_\theta (\theta, w), e_\theta (\theta, w), V_\theta (\theta, w),$ and $S_\theta (\theta, w)$ follows.

### A.10 Proof of Proposition 3

By Proposition 2, we have $v_L = 0$ and $w_H \leq w_L$. Lemmas 1 and 6 imply that $p_H = 1$.

Since $(IC_L)$ is satisfied with equality by Assumption 2, we have $V(L, w_H) = p_L V(L, w_L)$. Since $V$ is log-submodular and $w_H \leq w_L$, we have $V(H, w_H)/V(L, w_H) \geq V(H, w_L)/V(L, w_L)$, and so $V(H, w_H) \geq p_L V(H, w_L)$. Hence $(IC_H)$ is satisfied whenever $w_H \leq w_L$.

Hence the principal’s problem becomes

$$\max_{w_H, w_L, w_H \geq w_L \geq 0} \mu_H (\pi (H, w_H) - W) + \mu_L p_L (\pi (L, w_L) - W)$$

subject to $V(L, w_H) = p_L V(L, w_L)$. Substituting the constraint yields (10).

### A.11 Proof of Proposition 4

By Proposition 2, we have $w_H \geq w_L$. Lemmas 1 and 6 imply that $p_L = 1$.

Suppose $w_H \neq w_L$ but $(IC_H)$ holds with strict inequality (and so $v_L = 0$). Since $(IC_L)$ is satisfied with equality, we have $p_H V(L, w_H) = V(L, w_L)$. Since $V$ is log-submodular and $w_H > w_L$, we have $V(H, w_H)/V(L, w_H) < V(H, w_L)/V(L, w_L)$, and so $p_H V(H, w_H) < p_L V(H, w_L)$. Hence $(IC_H)$ is not satisfied.

Hence $w_H \neq w_L$ implies $(IC_H)$ holds with equality. Since $w_H = w_L$ implies $p_H = p_L$ and
$v_L = 0$ from Lemma 6, regardless of $(w_H, w_L)$, $(IC_H)$ holds with equality.

$$p_H V (H, w_H) = V (H, w_L) + v_L;$$

$$p_H V (L, w_H) = V (L, w_L) + v_L.$$  

Substituting them into (4) yields (11).

Moreover, rearranging $(IC_H)$ and $(IC_L)$, the fixed payment is equal to

$$v_L = \frac{V (H, w_L) V (L, w_H) - V (L, w_L) V (H, w_H)}{V (H, w_H) - V (L, w_H)},$$

which is positive if and only if $w_H > w_L$ given $V$ log-submodular.

**A.12 Proof of Proposition 5**

We consider the following relaxed problem:

$$\max_{w_L, w_H} \mathbb{E}_\theta [p_\theta \cdot S (\theta, w_\theta) + (1 - p_\theta) \cdot W]. \tag{24}$$

For each $\theta$, $w_\theta = 1$ (“selling the project”) maximizes the social welfare: For each $w \in [0, 1]$

$$S(\theta, 1) = \max_e (q (e) - c (\theta, e)) \geq q (e (\theta, w)) - c (\theta, e (\theta, w)) = S(\theta, w).$$

Given $w_\theta = 1$, (24) is maximized by taking $p_\theta = 1$ if $S (\theta, 1) \geq W$ and $p_\theta = 0$ otherwise. Since $W < W^{MAX} = \max_{w_H} \pi (H, w_H) \leq \max_{w_H} S (H, w_H)$, we have $p_H = 1$; and $p_L = 1$ if $S (L, 1) > W$ and only if $S (\theta, 1) \geq W$.\textsuperscript{42}

We are left to verify that there exists $v_\theta$ such that $(w_\theta, p_\theta)$ is incentive compatible (note that $v_\theta$ does not affect the objective). By setting $v_L = (p_H - p_L) V (L, 1)$, it is straightforward to verify that both $(IC_H)$ and $(IC_L)$ are satisfied.

\textsuperscript{42}Since $S (\theta, 1) = W$ implies any $p_L \in [0, 1]$ is optimal, we will simply say $p_L = 1$ if and only if $S (\theta, 1) \geq W$.  

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Online Appendix – For Online Publication

B Outside Options for Agents

B.1 Proof of Proposition 6

We have $v_H = 0$ given Lemma 4. Since $(IC_L)$ binds by Assumption 4, the principal’s problem is

$$\max_{p_H, w_H, p_L, w_L, v_L} \mu_H p_H (\pi (H, w_H) - W) + \mu_L [p_L (\pi (L, w_L) - W) - v_L]$$

(25)

subject to

$$p_H [V (H, w_H) - V_H^O] \geq \max \left\{ p_L [V (H, w_L) - V_H^O] + v_L, 0 \right\}, \quad (IC_H)$$

$$p_L [V (L, w_L) - V_L^O] + v_L = p_H [V (L, w_H) - V_L^O]. \quad (IC_L)$$

Substituting $(IC_L)$ yields

$$\max_{p_H, w_H, p_L, w_L} \mu_H p_H (\pi (H, w_H) - W) + \mu_L \left\{ p_L [S (L, w_L) - W - V_L^O] - p_H [V (L, w_H) - V_L^O] \right\}$$

subject to

$$p_H [V (H, w_H) - V_H^O] \geq \max \left\{ p_L \left\{ [V (H, w_L) - V_H^O] - [V (L, w_L) - V_L^O] \right\} + p_H [V (L, w_H) - V_L^O], 0 \right\},$$

$$p_L [V (L, w_L) - V_L^O] \leq p_H [V (L, w_H) - V_L^O].$$

Note that the second constraint is equivalent to $v_L \geq 0$. 
B.1.1 Case 1: $V^O_H = V^O_L$

Suppose $V^O_H = V^O_L$. If $S(L, 1) - W - V^O_L \leq 0$, then reducing $p_L$ increases the objective. Note that, with $V^O_H = V^O_L$, the constraint becomes

$$p_H [V(H, w_H) - V^O_H] \geq \max \left\{ p_L [V(H, w_L) - V(L, w_L)] + p_H [V(L, w_H) - V^O_H], 0 \right\},$$
$$p_L [V(L, w_L) - V^O_L] \leq p_H [V(L, w_H) - V^O_L].$$

Hence, reducing $p_L$ always relaxes the constraint. So, $p_L = 0$ is optimal if $S(L, 1) - W - V^O_L \leq 0$.

On the other hand, if $S(L, 1) - W - V^O_L > 0$, then increasing $p_L$ from $p_L = 0$ improves the objective since, for sufficiently small $p_L$, the constraints are redundant. Therefore, $p_L = 0$ is optimal only if $S(L, 1) - W - V^O_L \leq 0$.

B.1.2 Case 2: $V^O_H > V^O_L$

If $V^O_H > V^O_L$, then even if $S(L, 1) - W - V^O_L \leq 0$, the principal cannot reduce $p_L$ to $p_L = 0$ if the constraint

$$p_H [V(H, w_H) - V^O_H] \geq \max \left\{ p_L \left\{ [V(H, w_L) - V^O_H] - [V(L, w_L) - V^O_L] \right\} + p_H [V(L, w_H) - V^O_L], 0 \right\}$$

is binding. Note that the term multiplied to $p_L$, $[V(H, w_L) - V^O_H] - [V(L, w_L) - V^O_L]$, can be negative since $V^O_H > V^O_L$.

On the other hand, suppose $S(L, 1) - W - V^O_L > 0$ and $p_L = 0$ is incentive compatible:

$$p_H [V(H, w_H) - V^O_H] \geq \max \left\{ p_H [V(L, w_H) - V^O_L], 0 \right\}.$$
1. \( V(L, w_H) - V^O_L < 0 \). In this case,

\[
\max \left\{ p_L \left\{ [V(H, w_L) - V^O_H] - [V(L, w_L) - V^O_L] \right\} + p_H \left\{ V(L, w_H) - V^O_L \right\}, 0 \right\} = 0
\]

for sufficiently small \( p_L \). Hence, for sufficiently small \( p_L \), constraint \((IC_H)\) is redundant. The same proof as Proposition 1 then implies \( p_L > 0 \) at the optimum.

2. \( V(L, w_H) - V^O_L \geq 0 \). In this case, \((IC_H)\) at \( p_L = 0 \) is equivalent to

\[
V(H, w_H) - V(L, w_H) \geq V^O_H - V^O_L \geq 0.
\]

Suppose \( V(H, w_H) - V(L, w_H) > V^O_H - V^O_L \). Then, for sufficiently small \( p_L \), constraint \((IC_H)\) is redundant. The same proof as Proposition 1 implies \( p_L > 0 \) at the optimum. Suppose instead that \( V(H, w_H) - V(L, w_H) = V^O_H - V^O_L \). Since the first order condition for \( w_H \) is satisfied for the principal’s problem, the first order effect of increasing \( w_H \) on the principal’s objective is 0. Yet, Assumption 1 implies that the first order effect on \( V(H, w_H) - V(L, w_H) \) is positive; so, increasing \( w_H \) slightly makes \((IC_H)\) slack. With this slackness, constraint \((IC_H)\) is redundant for sufficiently small \( p_L \). Hence, the same proof as Proposition 1 implies that there is a first-order gain of increasing \( w_H \) and \( p_L \) for the principal.

**B.2 Retention of Both Types: \( V^O_H = V^O_L \)**

If \( V^O_H = V^O_L \), then we can show that, show the following: If \( V(\theta, w) \) is log-submodular, then \( V(\theta, w) - V^O_\theta \) is also log-submodular:

**Lemma 7** Suppose \( V^O_H = V^O_L = V^O \). If \( V(\theta, w) \) is log-submodular, then \( V(\theta, w) - V^O_\theta \) is also log-submodular.

Recall that the log-submodularity of \( V(\theta, w) \) implies that the higher type values retention probabilities more than monetary rewards. Since both types have the same outside option,
the outside option is relatively less attractive to type $H$. Hence this type values retention probabilities even more compared to type $L$, with the same outside options between the two types.

A similar relationship holds for $S(\theta, w) - V^O_\theta - W$:

**Lemma 8** Suppose $V^O_H = V^O_L = V^O$. If $S(\theta, w) - W$ is log-submodular, then $S(\theta, w) - V^O_\theta - W$ is also log-submodular. However, if $S(\theta, w) - W$ is log-supermodular, $S(\theta, w) - V^O_\theta - W$ may not be log-supermodular.

These lemmas follow from the following lemma, noting that $F_{w\theta}(\theta, w) \geq 0$, $F_w(\theta, w) \geq 0$, $F_\theta(\theta, w) \geq 0$ for each $F \in \{V, S\}$ from (14):

**Lemma 9** Consider any $F(\theta, w)$ with $F_{w\theta}(\theta, w) \geq 0$, $F_w(\theta, w) \geq 0$, $F_\theta(\theta, w) \geq 0$, and $W \geq 0$. If $F(\theta, w)$ is log-submodular, then $F(\theta, w) - W$ is also log-submodular as long as $F(\theta, w) - W \geq 0$. However, even if $F(\theta, w)$ is log-supermodular, $F(\theta, w) - W$ may not be log-supermodular.

**Proof.** For simplicity, we assume log $F(\theta, w)$ is twice-differentiable. Otherwise, we can take derivative of $(F(\theta, w) - W)(F(\theta', w') - W) - (F(\theta, w') - W)(F(\theta', w) - W)$ with respect to $W$ to derive the same result.

By the standard argument, $F(\theta, w) - W$ is log-supermodular (or submodular) if this cross derivative is non-negative (or non-positive). Hence we are left to show that

$$\frac{d}{dW}\frac{d^2}{d\theta dw} \log (F(\theta, w) - W) \leq 0$$

if $\frac{d^2}{d\theta dw} \log (F(\theta, w) - W) \leq 0$.

We have

$$\frac{d^2}{d\theta dw} \log (F(\theta, w) - W) = \frac{F_{w\theta}(\theta, w)[F(\theta, w) - W] - F_w(\theta, w)F_\theta(\theta, w)}{[F(\theta, w) - W]^2}$$

O.4
and
\[
\frac{d}{dW} \frac{d^2}{d\theta dw} \log \left( F(\theta, w) - W \right) = \frac{F_{w\theta}(\theta, w) [F(\theta, w) - W] - 2F_w(\theta, w) F_{\theta}(\theta, w)}{[F(\theta, w) - W]^3}.
\]

Given \( F_{\theta w}(\theta, w) \geq 0 \) and \( F(\theta, w) - W \geq 0 \), for \( \frac{d}{dW} \frac{d^2}{d\theta dw} \log \left( F(\theta, w) - W \right) \leq 0 \), it suffices to show that
\[
F_{w\theta}(\theta, w) [F(\theta, w) - W] \leq F_w(\theta, w) F_{\theta}(\theta, w),
\]
which is implied by \( \frac{d^2}{d\theta dw} \log \left( F(\theta, w) - W \right) \leq 0 \), as desired. ■

Lemma 7 implies that \( V(\theta, w) - V_\theta \) is log-submodular if \( V(\theta, w) \) is log-submodular. As in Proposition 2, we can characterize the optimal contract as follows:

**Proposition 11** Suppose \( V^O_H = V^O_L = V^O \). Consider the case in which the condition of Proposition 6 is not satisfied. Suppose that \( V(\theta, w) \) is log-submodular and \( S(\theta, w) - V^O_\theta - W \) is regular. The optimal contract has the following properties:

1. **(Principal-Agent RVE Co-Movement)** If \( S(\theta, w) - V^O_\theta - W \) is also log-submodular, then
   \[
   v_L = 0, \ w_L \geq w_H, \ 1 = p_H \geq p_L > 0.
   \]

2. **(Principal-Agent RVE Counter-Movement)** If \( S(\theta, w) - V^O_\theta - W \) is log-supermodular, then \((IC_H)\) holds with equality, and
   \[
   v_L \geq 0, \ w_H \geq w_L, \ 1 = p_L \geq p_H > 0.
   \]

   In addition, \( v_L > 0 \) if and only if \( w_H \neq w_L \).

**Proof.** The same as Proposition 2. ■

We can also derive analogous propositions to Propositions 3 and 4. We omit the details since it follows from the direct substitution of binding constraints.
B.3 Retention of Both Types: $V_H^O > V_L^O$

If $V_H^O > V_L^O$, then it is possible that type H prefers the outside option $V_H^O$ to type L’s contract. The following proposition characterizes when this is the case:

**Proposition 12** Suppose $V_H^O > V_L^O$. Consider the case in which the solution of (25) has $p_L > 0$. Define $w_H^O$ and $w_L^O$ such that

$$w_H^O = \arg \max_{w_H: V(H, w_H) \geq V_H^O} \frac{S(H, w_H) - V(H, w_H) - W}{V(L, w_H) - V_L^O};$$

and

$$w_L^O = \arg \max_{w_L} \frac{V(L, w_L) - V_L^O}{V(L, w_H^O) - V_L^O} (S(H, w_H^O) - V(H, w_H^O) - W) + S(L, w_L) - V(L, w_L) - W.$$ 

The optimal contract satisfies $w_H = w_H^O$, $w_L = w_L^O$, $p_H = \frac{V(L, w_L^O) - V_L^O}{V(L, w_H^O) - V_L^O}$, $p_L = 1$, and $v_L = 0$ if and only if we have $w_H^O \geq w_L^O$ together with

$$0 \geq V(H, w_L^O) - V_H^O,$$

$$0 \leq \frac{V(L, w_L^O) - V_L^O}{V(L, w_H^O) - V_L^O} (S(H, w_H^O) - V(H, w_H^O) - W) + S(L, w_L^O) - V(L, w_L^O) - W.$$ 

Moreover, in this case, type H prefers the outside option $V_H^O$ to type L’s contract.

**Proof.** We guess that type H prefers the outside option:

$$p_L [V(H, w_L) - V_H^O] + v_L < 0. \quad (26)$$

In such a case, since $C_L$ does not affect $(IC_H)$, we have $v_L = 0$ (otherwise, reduce $v_L$ and increase $w_L$). Since $(IC_L)$ is binding, the principal’s problem is

$$\max_{p_H, w_H, p_L, w_L} p_H (S(H, w_H) - V(H, w_H) - W) + p_L (S(L, w_L) - V(L, w_L) - W)$$

O.6
subject to

\[ V(H, w_H) - V_H^O \geq 0. \] (PC_H)

\[ p_L [V(L, w_L) - V_L^O] = p_H [V(L, w_H) - V_L^O]. \] (IC_L)

Moreover, if (26) is true, then we have \( 0 \geq V(H, w_L) - V_H^O \). Hence, at optimal, we have to have \( w_H \geq w_L \). This implies that \( p_H \leq p_L = 1 \).

Substituting (IC_L) and \( p_L = 1 \), we obtain

\[
\max_{w_H, w_L} \frac{V(L, w_L) - V_L^O}{V(L, w_H) - V_L^O} (S(H, w_H) - V(H, w_H) - W) + S(L, w_L) - V(L, w_L) - W
\]

subject to \( V(H, w_H) \geq V_H^O \). We ignore the constraint \( p_H \leq 1 \) since we know that, if type \( H \) prefers the outside option \( V_H^O \) to type \( L \)’s contract – if the conjecture is correct –, then we have \( w_H \geq w_L \) given \( V(H, w_H) \geq V_H^O \) (type \( H \) prefers \( C_H \) to the outside option), so (IC_L) implies \( p_H \leq 1 \).

Hence,

\[ w_H^O = \arg \max_{w_H} \frac{S(H, w_H) - V(H, w_H) - W}{V(L, w_H) - V_L^O}, \]

subject to \( V(H, w_H) \geq V_H^O \). Given the solution \( w_H^O \),

\[ w_L^O = \arg \max_{w_L} \frac{V(L, w_L) - V_L^O}{V(L, w_H^O) - V_L^O} (S(H, w_H^O) - V(H, w_H^O) - W) + S(L, w_L) - V(L, w_L) - W. \]

The guess is verified if and only if \( V_H^O \) is preferred to \( C_L \) and \( p_L = 1 \) is optimal, that is,

\[ 0 \geq V(H, w_L^O) - V_H^O; \]

\[ 0 \leq \frac{V(L, w_L^O) - V_L^O}{V(L, w_H^O) - V_L^O} (S(H, w_H^O) - V(H, w_H^O) - W) + S(L, w_L^O) - V(L, w_L^O) - W. \]

If the condition in the above proposition is not satisfied, then, at the optimal contract, the type \( H \) agent prefers \( C_L \) to the outside option. In this case, the analysis is the same.
as above, replacing $V(\theta, w)$ with $V(\theta, w) - V_\theta$. Since it is possible that $V(\theta, w) - V_\theta$ is log-supermodular even if $V(\theta, w)$ is log-submodular,\(^4\) the full characterization takes the following form:

**Proposition 13** Suppose $V_H^O > V_L^O$. Consider the case in which the solution of (25) has $p_L > 0$ and the condition of Proposition 12 is not satisfied. If both $V(\theta, w) - V_\theta$ and $S(\theta, w) - V^O - W$ are regular, then the optimal contract has the following properties:

1. **(RVE Co-Movement with Log-submodular $V(\theta, w) - V_\theta$)** If both $V(\theta, w) - V_\theta$ and $S(\theta, w) - V^O - W$ are log-submodular, then $v_L = 0$, $w_L \geq w_H$, and $1 = p_H \geq p_L > 0$.

2. **(RVE Co-Movement with Log-supermodular $V(\theta, w) - V_\theta$)** If both $V(\theta, w) - V_\theta$ and $S(\theta, w) - V^O - W$ are log-supermodular, then $v_L = 0$, $w_H \geq w_L$, and $1 = p_L \geq p_H > 0$.

3. **(RVE Counter-Movement with Log-submodular $V(\theta, w) - V_\theta$)** If $V(\theta, w) - V_\theta$ is log-submodular and $S(\theta, w) - V^O - W$ is log-supermodular, then $v_L \geq 0$, $w_H \geq w_L$, and $1 = p_L \geq p_H > 0$. In addition, $v_L > 0$ if and only if $w_H \neq w_L$.

4. **(RVE Counter-Movement with Log-supermodular $V(\theta, w) - V_\theta$)** If $V(\theta, w) - V_\theta$ is log-supermodular and $S(\theta, w) - V^O - W$ is log-submodular, then $v_L \geq 0$, $w_L \geq w_H$, $1 = p_H \geq p_L > 0$. In addition, $v_L > 0$ if and only if $w_H \neq w_L$.

**Proof.** The same as Proposition 2. \[\blacksquare\]

Moreover, we can obtain a similar characterization as in Propositions 3 and 4 for each of the four cases. We omit the details since the result follows from the direct substitution of binding constraints.

\(^4\)On the contrary to Lemma 9, even if the higher type values retention probabilities compared to the low type without outside options, the $H$ type may value monetary rewards, compared to the $L$ type, if the $H$-type outside option is very high and he would like to obtain the outside option.
C Model with More Than Two Outcomes

Assumption 5 implies \((IC_L)\) binds. As in Proposition 16, \(p_L = 0\) if and only if \(S(L, w) < W\) with \(w = (y)_{y \in Y}\) (giving all the output to the agent). The proof is the same as Proposition 16 and so is omitted. Hence, we focus on the case in which \((IC_L)\) binds and \(p_L > 0\).

C.1 A General Condition

When we prove Proposition 2, we consider the principal’s deviation to offer the same contract \(w_\theta\) to both types. The non-profitability of this deviation amounts to

\[
\begin{align*}
    p_H [S(H, w_H) - W] &\geq p_L [S(H, w_L) - W]; \\
    p_H [S(L, w_H) - W] &\leq p_L [S(L, w_L) - W].
\end{align*}
\]

That is,

\[
(S(H, w_H) - W)(S(L, w_L) - W) - (S(H, w_L) - W)(S(L, w_H) - W) \geq 0. \tag{27}
\]

By the same proof as Lemma 2, we can show that \(v_L > 0\) implies both \((IC_H)\) and \((IC_L)\) are binding. Manipulating them yields

\[
v_L = p_L \frac{V(H, w_L) V(L, w_H) - V(L, w_L) V(H, w_H)}{V(H, w_H) - V(L, w_H)}.
\]

Hence

\[
V(H, w_L) V(L, w_H) - V(L, w_L) V(H, w_H) > 0 \text{ if and only if } v_L > 0. \tag{28}
\]

If \(|Y| = 2\), then the log-submodularity/supermodularity of \(S(\theta, w) - W\) and \(V(\theta, w)\) with \(w = w(1) - w(0)\) ensures that (27) and (28) imply that \(v_L > 0\) only if \(S(\theta, w) - W\) is log-supermodular given log submodular \(V(\theta, w)\). To see why, given log submodular \(V(\theta, w)\),
we have

$$[(S(H, w_H) - W)(S(L, w_L) - W) - (S(H, w_L) - W)(S(L, w_H) - W)]$$

$$\times [V(H, w_H)V(L, w_L) - V(H, w_L)V(L, w_H)] \geq 0$$

for each $w_L, w_H$ if and only if $S(\theta, w) - W$ is log-submodular. The analogous condition with multiple outcomes is that

$$[(S(H, w_H) - W)(S(L, w_L) - W) - (S(H, w_L) - W)(S(L, w_H) - W)]$$

$$\times [V(H, w_H)V(L, w_L) - V(H, w_L)V(L, w_H)]$$

$$\geq 0 \text{ for each } w_L, w_H. \quad (29)$$

We obtain a result analogous to the co-moving RVE of Proposition 2:

**Proposition 14** If (29) holds, then $v_L = 0$.

**Proof.** The same as Proposition 2. ■

### C.2 A Specific Condition

One may wonder if there is a condition more closely related to the log-submodularity/supermodularity of $S(\theta, w) - W$ and $V(\theta, w)$. To this end, we first show that Assumption 6 implies that the $H$ type is compensated only after the highest outcome:

**Lemma 10** Given Assumption 6, we have $w_H = (0, \ldots, 0, w_H)$ in the optimal contract.

As seen in Section 6.2, the first condition of Assumption 6 is monotone hazard rate condition – the principal would like to compensate the agent only after the highest outcome without adverse selection – and the second condition (monotone likelihood ratio condition) ensures that the low type’s incentive to mimic the high type is decreased if the high type is compensated only after the highest outcome.
Proof. Let $w^n_H$ be the wage after outcome $y_n$. If there exist $n$ and $n'$ such that $n > n'$ and $w^n_H, w^{n'}_H > 0$, suppose the principal increases $w^n_H$ by one unit, which increases the type $H$’s payoff by $q(y_n|e_H)$. To keep the $H$ type indifferent, the principal reduces $w^{n'}_H$ by $q(y_n|e_H)/q(y_{n'}|e_H)$. With constant $e_H$, the principal would be indifferent to this perturbation. Yet, the monotone hazard rate condition implies that the $H$ type increases his effort, and so the principal’s payoff is increasing.

For the $L$ type, by the Envelope Theorem, the gain from imitating the $H$ type changes by

$$q(y_n|e_{LH}) - \frac{q(y_n|e_H)}{q(y_{n'}|e_H)}q(y_{n'}|e_{LH}),$$

where $e_{LH}$ is the optimal effort of the $L$ type when he takes contract $C_H$. Since $e_H > e_{LH}$, the monotone likelihood ratio condition implies that the above gain is negative. Hence $(IC_L)$ gets relaxed. This implies that we have $w^n_H > 0$ only for $n = |Y|$. ■

As mentioned in Section 6.2, it is not always the case that the $L$ type is compensated only after the highest outcome. However, we can still derive the condition which depends only on $V_{|Y|}(\theta, w)$ and $S_{|Y|}(\theta, w) - W$ (the agent’s value and social welfare, assuming that the agent is compensated only after the highest outcome), as seen in Proposition 7:

**Proposition 15 (Proposition 7)** Suppose Assumptions 5 and 6 are satisfied and (12) holds. In addition, $S(L, w) < W$ with $w = (y)_{y \in Y}$. Moreover, suppose $V_{|Y|}(\theta, w)$ is log-submodular and $S_{|Y|}(\theta, w) - W$ is regular. Then $v_L \geq 0$ only if $S_{|Y|}(\theta, w) - W$ is log-supermodular.

**Proof.** In Appendix C.3. ■

**C.3 Proof of Proposition 7**

We prove that $v_L > 0$ only if $S_{|Y|}(\theta, w)$ is log-supermodular. To see why, suppose $v_L > 0$. Then, since $(IC_H)$ is binding (otherwise, reducing $v_L$ and increasing $w_{|Y|}$ to keep the $L$ type
indifferent would be a profitable deviation), we have

\[ p_H V(H, w_H) = p_L V(H, w_L) + v_L; \]
\[ p_H V(L, w_H) = p_L V(L, w_L) + v_L. \]

Hence,

\[ \frac{v_L}{p_L} = \frac{V(H, w_L) V(L, w_H) - V(L, w_L) V(H, w_H)}{V(H, w_H) - V(L, w_H)} > 0, \]

which implies

\[ V(H, w_L) V(L, w_H) - V(L, w_L) V(H, w_H) > 0. \]

Since \( \Delta V(H, w) \geq 0 \) and \( V(L, w(L, w_L)) = V(L, w_L) \), this implies

\[ V(H, w(L, w_L)) V(L, w_H) - V(L, w(L, w_L)) V(H, w_H) > 0, \]

or

\[ V_{[Y]}(H, w(L, w_L)) V_{[Y]}(L, w_H) - V_{[Y]}(L, w(L, w_L)) V_{[Y]}(H, w_H) > 0. \]

Thus, \( w(L, w_L) < w_H \) by the log-submodularity of \( V_{[Y]}(\theta, w) \) (recall that \( w(L, w_L) = (0, ..., 0, w(L, w_L)) \)).

The principal’s deviation of offering \( w(L, w_L) \) to both types is not profitable, so

\[ p_L (S_{[Y]}(H, w(L, w_L)) - V(H, w(L, w_L)) - W) - v_L \leq p_H (S_{[Y]}(H, w_H) - V_{[Y]}(H, w_H) - W). \]

Since \( w(L, w_L) \) and \( w_H \) have positive rewards only after \( y_{[Y]} \), it follows that \( S_{[Y]}(H, w(L, w_L)) = S(H, w(L, w_L)) \) and \( S_{[Y]}(H, w_H) = S(H, w_H) \). By \((IC_H)\),

\[ p_L V(H, w_L) + v_L \leq p_H V_{[Y]}(H, w_H). \]
From $V(H, w_L) = V(H, w(L, w_L)) - \Delta V(H, w_L)$, we obtain

$$p_L(S|Y| (H, w(L, w_L)) - W - \Delta V(H, w_L)) \leq p_H(S|Y| (H, w_H) - W) .$$

(30)

Similarly, since the principal’s deviation of offering $w_H$ to both types is not profitable, we have

$$p_H(S(L, w_H) - W) \leq p_L(S(L, w_L) - W) .$$

Moreover, since $S(L, w_H) = S|Y| (L, w_H)$ and $S(L, w_L) = S(L, w(L, w_L)) - \Delta S(L, w_L) = S|Y| (L, w(L, w_L)) - \Delta S(L, w_L)$, we also have

$$p_H(S|Y| (L, w_H) - W) \leq p_L(S|Y| (L, w(L, w_L)) - \Delta S(L, w_L) - W) .$$

(31)

Inequalities (30) and (31) imply

$$(S|Y| (L, w_H) - W) (S|Y| (H, w(L, w_L)) - W - \Delta V(H, w_L))$$

$$\leq (S|Y| (H, w_H) - W) (S|Y| (L, w(L, w_L)) - \Delta S(L, w_L) - W) .$$

The inequality $S|Y| (L, w_H) - W \leq S|Y| (H, w_H) - W$, together with (12), implies

$$(S|Y| (H, w_H) - W) \Delta S(L, w_L) \geq (S|Y| (L, w_H) - W) \Delta V(H, w_L) .$$

Hence,

$$(S|Y| (L, w_H) - W) (S|Y| (H, w(L, w_L)) - W)$$

$$\leq (S|Y| (H, w_H) - W) (S|Y| (L, w(L, w_L)) - W) .$$

Finally, $w(L, w_L) < w_H$ implies that $S|Y| (\theta, w) - W$ is log-supermodular.
D  Endogenous Outside Option \(W\)

D.1 Proof of Lemma 5

Suppose \((IC_L)\) is not binding. Let \(C^*_\theta = (p^*_\theta, w^*_\theta, v^*_\theta)\) be the optimal contract for type \(\theta\); and let \(J^*_\theta\) be the principal’s profit from type \(\theta\):

\[
J^*_\theta = p^*_\theta q(e(L, w^*_L)) (1 - w^*_L) + (1 - p^*_\theta)(\bar{J} - \kappa) - v^*_\theta.
\]

Note that \(J^*_H \geq J^*_L\), since offering \(C_H = C^*_L\) is incentive compatible.

Then,

\[
\frac{d}{dw_L} q(e(L, w_L)) (1 - w_L) \bigg|_{w_L=w^*_L} \geq 0
\]

since otherwise the principal would decrease \(w^*_L\) without violating \((IC_H)\). Hence, \(w^*_L \leq w^*_L^{PB} \equiv \max_{w_L \geq 0} q(e(L, w_L)) (1 - w_L)\), and so

\[
q(e(L, w^*_L)) (1 - w^*_L) \leq \pi^*_L \equiv \max_{w_L \geq 0} q(e(L, w_L)) (1 - w_L).
\]

Since offering \(p = 1, w = w^*_L^{PB}, v = 0\) to both agent is possible, we have

\[
\bar{J} > \mu_H q(e(H, w^*_L^{PB})) (1 - w^*_L^{PB}) + \mu_L \pi^*_L.
\]

The strict inequality follows from Assumption 1. Then, there exists \(\varepsilon > 0\) such that

\[
\bar{J} \geq q(e(L, w^*_L)) (1 - w^*_L) + \varepsilon.
\]

Therefore, with \(\kappa \leq \varepsilon\), the principal would decrease \(p_L\) if \((IC_L)\) were not binding.

D.2 Proof of Proposition 8

Proposition 8 results from the following three propositions, which characterize each of the three cases stated in the proposition:
Proposition 16  In the contract in which the $L$ type is never retained, $p_L = 0$, $p_H = 1$, $v_L > 0$, and the optimal $w_H$ maximizes

$$J := \max_{w_H} q(e(H, w_H))(1 - w_H) - \frac{\mu_L}{\mu_H} (V(L, w_H) + \kappa).$$  \hspace{1cm} (33)$$

This contract is optimal if and only if $S(L, 1) - (J - \kappa) \leq 0$.

Proof. By the same proof as Proposition 1, $p_L = 0$ is optional if and only if $S(L; 1) \leq 0$. We guess $p_L = 0$ is optimal. That is, $S(L, 1) - (W - \kappa) \leq 0$. Then, at the fixed point, the value is

$$\bar{J} = \max_{w_H} \mu_H q(e(H, w_H))(1 - w_H) + \mu_L \left[ (\bar{J} - \kappa) - V(w_H, L) \right]$$

\[
\Leftrightarrow
\]

$$\bar{J} = J := \max_{w_H} q(e(H, w_H))(1 - w_H) - \frac{\mu_L}{\mu_H} (V(L, w_H) + \kappa).$$

This guess is verified if and only if $S(L, 1) - (J - \kappa) \leq 0$. \hfill \blacksquare

Proposition 17  If $V(\theta, w)$ is log-submodular, $S(\theta, w) - W$ is log-submodular, $(IC_L)$ binds, and the condition of Proposition 16 does not hold, then $v_L = 0$, $1 = p_H \geq p_L$ and $w_L \geq w_H$. In particular, $w_H$ and $w_L$ solve

$$\max_{w_H, w_L, w_L \geq w_H \geq 0} \frac{\mu_H \pi(H, w_H) + \mu_L \left( \frac{V(L, w_H)}{V(L, w_L)} \pi(L, w_L) - \frac{V(L, w_L) - V(L, w_H)}{V(L, w_L)} \kappa \right)}{\mu_H + \mu_L \frac{V(L, w_H)}{V(L, w_L)}}$$

and $p_L = \frac{V(L, w_H)}{V(L, w_L)}$.

Proof. Suppose $S(\theta, w) - W$ is log-submodular for each $W$. Then, by the same proof as Proposition 2, we have $w_H \leq w_L$, $p_H \geq p_L$, and $v_L = 0$. Moreover, $(IC_H)$ is not binding.
Hence, the problem becomes
\[
\bar{J}(W) = \max_{w_H, w_L, w_L \geq w_H \geq 0} \mu_H \pi(H, w_H) + \mu_L \frac{V(H, w_H)}{V(L, w_L)} \pi(L, w_L) + \frac{V(L, w_L) - V(L, w_H)}{V(L, w_L)} (W - \kappa).
\]

At the fixed point,
\[
\bar{J} = \frac{\mu_H \pi(H, w_H) + \mu_L \left( \frac{V(L, w_L)}{V(H, w_H)} \pi(L, w_L) - \frac{V(L, w_L) - V(L, w_H)}{V(L, w_L)} \kappa \right)}{\mu_H + \mu_L \frac{V(L, w_H)}{V(L, w_L)}}.
\]

By dynamic programming, it follows that the principal maximizes \((34)\). ⊡

**Proposition 18** If \(V(\theta, w)\) is log-submodular, \(S(\theta, w) - W\) is log-supermodular, \((IC_L)\) binds, and the condition of Proposition 16 does not hold, then \(v_L \geq 0\), \(p_H \leq p_L = 1\) and \(w_L \leq w_H\).

In particular, \(w_H\) and \(w_L\) solve
\[
\bar{J}(W) = \max_{w_H, w_L, w_L \geq w_H \geq 0} \frac{\mu_H}{\mu_H V(H, w_H) - V(L, w_L)} \pi(H, w_H) - \left(1 - \frac{V(H, w_L) - V(L, w_H)}{V(H, w_H) - V(L, w_H)} \kappa\right)
\]
\[
+ \mu_L \frac{\pi(L, w_L) - V(L, w_H) + V(L, w_L)}{\mu_H V(H, w_H) - V(L, w_L)} + \mu_L.
\]

and \(p_H = \frac{V(H, w_L) - V(L, w_L)}{V(H, w_H) - V(L, w_L)}\).

**Proof.** Suppose \(S(\theta, w) - W\) is log-supermodular for each \(W\). Then, by the same proof as Proposition 2, we have \(w_H \geq w_L\), \(p_H \leq p_L\), and \(v_L \geq 0\). Moreover, both \((IC_H)\) and \((IC_L)\) are binding. Substituting them into the objective, the problem becomes
\[
\bar{J}(W) = \max_{w_H, w_L, w_L \geq w_H \geq 0} \frac{\mu_H}{\mu_H V(H, w_H) - V(L, w_L)} \pi(H, w_H)
\]
\[
+ \mu_H \left(1 - \frac{V(H, w_L) - V(L, w_H)}{V(H, w_H) - V(L, w_H)} \kappa\right)
\]
\[
+ \mu_L \left(\pi(L, w_L) - V(L, w_H) + V(L, w_L)\right).
\]
At the fixed point,

\[
\bar{J} = \mu_H \frac{V(H,w_L) - V(L,w_L)}{V(H,w_H) - V(L,w_H)} \pi(H,w_H) - \left( 1 - \frac{V(H,w_L) - V(L,w_L)}{V(H,w_H) - V(L,w_H)} \right) \kappa \nu_H \frac{V(H,w_L) - V(L,w_L)}{V(H,w_H) - V(L,w_H)} + \mu_L \left( \frac{V(H,w_L) - V(L,w_L)}{V(H,w_H) - V(L,w_H)} \right)
\]

(35)

By dynamic programming, the principal maximizes (35).

E Proof of Proposition 9

By the same proof as Proposition 1, \( p_1 = 0 \) and \( v_1 > 0 \) if \( W > S(\theta_1, 1) \). We first derive the lower bound of \( W \) and then derive the upper bound of \( S(\theta_1, 1) \).

Suppose the principal offers \( (p_n, w_n, v_n) = \left( 1, \frac{\bar{c}}{q_1}, 0 \right) \) and \( (p_i, w_i, v_i) = \left( 0, 0, V(\theta_{n-1}, \frac{\bar{c}}{q_1}) \right) \) for each \( i \neq n \). Since \( IC_{n-1,n} \) is binding and \( \theta_{n-1} \) is the second highest type, each type \( i \leq n - 1 \) prefers \( (p_i, w_i, v_i) \) to \( (p_n, w_n, v_n) \). Hence, this contract is incentive compatible. Moreover, since \( c_e(\theta_n, \frac{1}{q_1}) \leq \bar{c} \), we have \( q\left(e\left(\theta_n, \frac{\bar{c}}{q_1}\right)\right) = 1 \) and so the principal’s profit is no less than

\[
\bar{J}(W) \geq \mu_{\theta_n} \left( 1 - \frac{\bar{c}}{q_1} \right) + \left( 1 - \mu_{\theta_n} \right) \left( -\kappa - V(\theta_{n-1}, \frac{\bar{c}}{q_1}) + W \right) \]

\[
\geq \mu_{\theta_n} \left( 1 - \frac{\bar{c}}{q_1} \right) + \left( 1 - \mu_{\theta_n} \right) \left( -\kappa - \frac{\bar{c}}{q_1} + W \right).
\]

Since \( W = \bar{J}(W) \) in equilibrium, we have

\[
W \geq 1 - \frac{\left( 1 - \mu_{\theta_n} \right) \kappa + \frac{\bar{c}}{q_1}}{\mu_{\theta_n}}.
\]

On the other hand, since \( c_e(\theta_1, 0) \geq \underline{c} \), if \( \underline{c} > q_1 \), then we obtain \( S(\theta_1, 1) = q_0 < \bar{q} \). For
sufficiently small $\kappa$, $\bar{c}$, and $\bar{q}$,

$$W \geq 1 - \frac{(1 - \mu_{\theta_n}) \kappa + \frac{\bar{c}}{q_n}}{\mu_{\theta_n}} > \bar{q} > q_0 \geq S(\theta_1, 1),$$

and so $p_1 = 0$ and $v_1 > 0$, as desired.

F Complementary Analysis

We now provide the characterization when $V(\theta, w)$ is log-supermodular, or the case in which $V(\theta, w)$ or $S(\theta, w) - W$ are not regular. We focus on the base model since we can generalize the extensions in an analogous way. In the base model, only Propositions 2, 3, and 4 use log-submodularity of $V(\theta, w)$ and regularity of $S(\theta, w) - W$. Hence, we focus on these three results.

First, we consider the case in which $V(\theta, w)$ is log-supermodular and $S(\theta, w) - W$ is regular. The following proposition is the counterpart of Proposition 2.

Proposition 19 Consider the case in which $IC_L$ binds and the condition of Proposition 1 is not satisfied. Suppose $V(\theta, w)$ is log-supermodular and $S(\theta, w) - W$ is regular. Then, the optimal contract has the following properties:

1. **(RVE Co-Movement)** If $S(\theta, w) - W$ is also log-supermodular, then $v_L = 0$, $w_L \leq W_H$, and $1 = p_L \geq p_H > 0$.

2. **(RVE Counter-Movement)** If $S(\theta, w) - W$ is log-submodular, then $(IC_H)$ holds with equality and $v_L \geq 0$, $w_L \geq W_H$, and $1 = p_H \geq p_L > 0$.

Proof. Symmetric to Proposition 2. ■

We can derive the full characterization as in Propositions 3 and 4. The details are omitted since it is completely symmetric to the case with log-submodular $V$.

If $V(\theta, w)$ or $S(\theta, w) - W$ is not regular, we can still consider log-supermodularity and log-submodularity at the optimal contract: Let $F(\theta, w) \in \{V(\theta, w), S(\theta, w) - W\}$. We say
$F$ is locally log-supermodular (or log-submodular) around the optimal contract if, given the wages $(w^*_H, w^*_L)$ at the optimal contract, for each $\theta, \theta' \in \{H, L\}$ and $w, w' \in \{w^*_H, w^*_L\}$, the following holds:

$$F(\theta, w) F(\theta', w') - F(\theta', w) F(\theta, w') \geq \langle 0$$

if and only if $(\theta - \theta')(w - w') \geq \langle 0$.

Note that, for any $F$ and $(w^*_H, w^*_L)$, we can always have either $F$ locally log-supermodular or $F$ locally log-submodular. The similar characterization as in Propositions 2 and 19 holds with this local concept.

### F.1 Numerical Illustration

We offer a numerical example with log-supermodular $V$. Suppose $c(e, \theta) = \frac{e^2}{2q}$. Then, the first order condition for $e$ is $e = \theta w q_1$. This implies

$$V(\theta, w) = q_0 w + \theta (q_1 w)^2, \pi(\theta, w) = (10 - w)(q_0 + \theta (q_1)^2 w),$$

and

$$S(\theta, w) = 10q_0 + w \left(10 - \frac{w}{2}\right) \theta (q_1)^2 - W.$$

Some algebra shows that

$$\frac{d^2 \log V(\theta, w)}{dwd\theta} = \frac{\frac{1}{2} q_0 (q_1 w)^2}{(q_0 w + \frac{\theta}{2} (q_1 w)^2)^2} \geq 0;$$

$$\frac{d^2 \log S(\theta, w)}{dwd\theta} = \frac{(10 - w)(q_1)^2 (10q_0 - W)}{(10q_0 + w \left(10 - \frac{w}{2}\right) \theta (q_1)^2 - W)^2}.$$
Hence, \( V(\theta, w) \) is log-supermodular; and \( S(\theta, w) \) is log-supermodular (or log-submodular) if and only if \( 10q_0 \geq W \) (\( 10q_0 \leq W \), respectively).

Suppose that \( 10q_0 \geq W \) and so \( S(\theta, w) \) is log-supermodular – the RVE co-movement case. Since we also have \( S(L, 10) \geq 10q_0 \geq W \), the optimal contract can possibly only take one of the following three forms:

1. Different contract for each type, and \( IC_L \) does not bind (so that \( w_H < w_L \) and \( v_L = 0 \));
2. Different contract for each type, and \( IC_H \) does not bind (so that \( w_L < w_H \) and \( v_L = 0 \));
3. Same contract for both types (\( w_L = w_H \) and \( v_L = 0 \)).

The result that \( v_L = 0 \) follows from the RVE co-movement. With log-supermodular \( S(\theta, w) - W \), the optimal contract features \( w_H \geq w_L \), given (8) and (9). Hence, the first case described above cannot happen. Moreover, since \( \pi(\theta, w) = (10 - w)(q_0 + \theta(q_1)^2 w) \), the principal would offer the wage \( w_B^{PB} = \max\{(-q_0 + 10\theta(q_1)^2)/2\theta(q_1)^2, 0\} \) if she knew the agent’s type. Since \( w_B^{PB} < w_L^{PB} \), the second case also cannot happen – it would be better to offer \( w_L = w_H \) rather than \( w_L < w_H \) (see Lemma 11 for the formal proof). Therefore, only the third case is optimal, and same contract is offered to both types of agents.

Suppose next that \( q_0 \leq W \) and so \( S(\theta, w) \) is log-submodular – the RVE counter-movement case. Then, the optimal contract can possibly only take one of the following three forms:

1. Different contract for each type, and \( IC_L \) does not bind (\( v_L = 0 \));
2. Only type \( H \) is retained (\( p_L = 0 \));
3. Same contract for both types;
4. Different contract for each type, both \( IC_H \) and \( IC_L \) bind (\( v_L > 0 \)).

For example, with \( \theta_L = 4, \theta_H = 5.5, q_0 = .2, \) and \( W = 3 \), for small \( q_1 \), the optimal contract has the first form outlined above. In particular, \( q_1 \leq .4 \) implies that \( IC_L \) does not

\[ \text{Here, we assume that no boundary condition on } q(e) \text{ is binding, but the same conclusion holds with binding constraints as well.} \]
bind and $v_L = 0$. For intermediate values of $q_1$ (for example, $q_1 = .6$ or $q_1 = .8$), the same contract is optimal and $v_L = 0$. Finally, with $q_1 = 1$, we have $p_H = 1$, $p_L = .76$, $w_H = 1.72$, $w_L = 2$, and $v_L = .044$.

Intuitively, for a small $q_1$, the principal would offer a high wage to type $L$ even if she knew the type ($w_{PB}^L$ is high since $q_1$ is small), so $IC_L$ is not binding. Yet, for a large $q_1$, $w_{PB}^H < w_{PB}^L$, so the principal would like to offer a higher wage to the low type. Since $q_1$ is sufficiently large, if $IC_L$ were not binding, then the principal would like to increase $w_H$. Hence, both $IC_H$ and $IC_L$ bind.

G With No Threat of Imitation from Type $L$

Assumption 2 implies $(IC_L)$ binding in the baseline problem. We now dispense with this assumption. Since $(IC_L)$ may not be binding, when we solve for the optimal contract, we proceed as follows:

1. Note that all the lemmas in Section 4.2 hold without Assumption 2. Hence, we can simplify the problem using them. In particular, $v_H = 0$.

2. Then, we ask if $(IC_L)$ is binding. Proposition 20 fully characterizes when it is not binding and what is the optimal contract if it is not binding.

3. If the condition of Proposition 20 does not hold, then $(IC_L)$ binds. Then, we proceed as in Propositions 1 and 2.\footnote{Use Proposition 19 instead of 2 if $V(\theta, w)$ is log-supernular. If $V(\theta, w)$ or $S(\theta, w) - W$ is not regular, then conduct a local analysis.} Note that none of the lemmas/propositions presented so far depends on Assumption 2 beyond the fact that $(IC_L)$ binds. Hence, they are valid, as they stand, if the condition of Proposition 20 does not hold.

Moreover, $W \geq S(L, 1)$ implies Assumption 2 since $S(L, 1) \geq \max_w \pi(L, w)$. Hence, Proposition 1 holds without the additional premise that the condition of Proposition 20 does not hold.
We now establish when constraint \((IC_L)\) binds. We first state the necessary condition for \((IC_L)\) not binding:

**Lemma 11** Constraint \((IC_L)\) does not hold with equality only if \(\pi(L, w_{PB}^L) > W\) and \(w_{PB}^H < w_{PB}^L\), where we define \(w_{PB}^L = \arg\max_{\theta \geq 0} \pi(\theta, w_\theta)\).

**Proof.** In Appendix G.1. ■

If \(\pi(L, w_{PB}^L) \leq W\), clearly the principal would reduce \(p_L\) if \((IC_L)\) were not binding. If \((IC_L)\) is not binding, \(\pi(L, w_{PB}^L) > W\), and \(w_{PB}^H \geq w_{PB}^L\), then the principal would like to both offer a higher wage to type \(H\) to address the moral hazard problem, and to also offer a higher probability of retention of this type, in order to address the adverse selection problem, since it satisfies \((IC_H)\). Moreover, \(v_L = 0\) since \((IC_L)\) is not binding. This is clearly not incentive compatible for the low type.

If constraint \((IC_L)\) does not bind, then type \(L\) strictly prefers the contract designed for him to the contract designed for type \(H\). Since the contract for type \(H\) is unattractive for type \(L\), the principal optimally retains type \(H\) at all times, \(p_H = 1\), and sets the fixed payment \(v_L\) to 0. Since there is no threat of type \(L\) pretending to be type \(H\), the only binding constraint is the one for type \(H\). This constrains how high \(p_L\) can be set. Hence, \(p_L \leq V(H, w_H) / V(H, w_L)\), and the optimal rewards \((w_H, w_L)\) and retention probability \(p_L\) are then determined by

\[
(\bar{w}_H, \bar{w}_L, \bar{p}_L) = \arg\max \mu_H \cdot \pi(H, w_H) + \mu_L \cdot p_L (\pi(L, w_L) - W). \tag{36}
\]

subject to \(w_H \geq 0, w_L \geq 0,\) and \(0 \leq p_L \leq V(H, w_H) / V(H, w_L)\).

**Proposition 20** When \(\pi(L, w_{PB}^L) > W\) and \(w_{PB}^H < w_{PB}^L\), \((p_H, w_H, v_H) = (1, \bar{w}_H, 0)\) and \((p_L, w_L, v_L) = (\bar{p}_L, \bar{w}_L, 0)\) is the optimal contract if and only if it satisfies \((IC_L)\), and in that case, \((IC_L)\) is not binding.

**Proof.** In Appendix G.2. ■
Intuitively, when \( \pi(L, w_{LB}^P) > W, w_{HB}^P < w_{LB}^P \), and constraint \((IC_L)\) is not binding, the principal’s actions to address the moral hazard problem – offering a lower wage to type \( H \) than to type \( L \) – and her actions to address adverse selection – offering a higher probability of retention to type \( H \) – go towards \( w_H < w_L \) and \( p_H > p_L \).

### G.1 Proof of Lemma 11

Suppose \((IC_L)\) holds with strict inequality. We assume \( w_{HB}^P \geq w_{LB}^P \) and will derive a contradiction.

We have \( v_L = 0 \) (otherwise the principal would reduce \( v_L \)) and \( \pi(L, w_L) \geq W \) (otherwise she would reduce \( p_L \) to obtain the outside option but \( p_L = v_L = 0 \) would violate \((IC_L)\)). Hence, the principal’s payoff is equal to

\[
\mu_H [p_H \pi(H, w_H) + (1 - p_H) W - v_H] + \mu_L [p_L \pi(L, w_L) + (1 - p_L) W - v_L]
\]

\[
\leq \mu_H [p_H \pi(H, w_H) + (1 - p_H) W - v_H] + \mu_L \pi(L, w_L) \text{ since } \pi(L, w_L) \text{ is no less than } W
\]

\[
\leq \mu_H \pi(H, w_H) + \mu_L \pi(L, w_L).
\]

The last inequality follows since offering \((p_L, w_L, v_L) = (1, w_L, 0)\) to both types is incentive compatible. Thus, \( \pi(H, w_H) \geq W \).

Given \( \pi(H, w_H) \geq W \), without loss, we have \( p_H = 1 \) (since increasing \( p_H \) does not violate \((IC_L)\) if this constraint holds with strict inequality). In addition, \( \pi_w(H, w_H) \leq 0 \) (since increasing \( w_H \) does not violate \((IC_L)\) if this constraint holds with strict inequality). This implies \( \frac{d}{dw} \pi(L, w_H) \leq 0 \) since we assume \( w_{HB}^P \geq w_{LB}^P \) and \( \pi \) is concave by (14).

Suppose \( p_L < 1 \). Then, we have to have \( w_L > w_H \) since otherwise \((IC_L)\) would be violated (recall \( v_H = v_L = 0 \) and \( p_H = 1 \)); however, \( \pi_w(L, w_L) < \pi_w(L, w_H) \leq 0 \) implies that reducing \( w_L \) improves \( \pi(L, w_L) \) (note that \((IC_L)\) still holds if it originally holds with strict inequality). Hence, \( p_L = 1 \).

Finally, given \( p_H = p_L = 1 \), Gottlieb and Moreira (2014) implies \( w_H = w_L \) and \( v_H = v_L = 0 \). Therefore \((IC_L)\) holds with equality.
G.2 Proof of Proposition 20

We start with

\[ \tilde{J}(W) = \max_{\{p_H,p_L,w_H,w_L,v_H\}} \mu_H [p_H (S(H,w_H) - V(H,w_H)) + (1 - p_H) W] \\
+ \mu_L [p_L (S(L,w_L) - V(L,w_L)) + (1 - p_L) W] \]

subject to \((IC_H)\): \(p_H V(H,w_H) \geq p_L V(H,w_L)\) and \((IC_L)\): \(p_L V(L,w_L) \geq p_H V(L,w_H)\).

Here, we use Lemmas 1 and 2 and substitute \(v_H = v_L = 0\).

Next, we will show that \(p_H = 1\). Since the problem is linear in \((p_H,p_L)\), we have \(p_H = 1\) or \(p_L = 1\). Suppose \(p_L = 1\) is optimal. Then we have \(S(L,w_L) - V(L,w_L) - W > 0\) since otherwise reducing \(p_L\) improves the objective and relaxes \((IC_H)\) (and we assume \((IC_L)\) is slack). Hence we have \(S(H,w_H) - V(H,w_L) - W > 0\). This implies \(S(H,w_H) - V(H,w_H) - W > 0\) (if \(S(H,w_H) - V(H,w_H) - W \leq 0\), then offering \(p_H = 1\) and \(w_H = w_L\) would improve the objective since type \(H\) brings a higher profit to the principal than type \(L\)). Hence, increasing \(p_H\) improves the objective and relaxes the constraint. Therefore, without loss, we can assume \(p_H = 1\). Now the problem becomes

\[ \max_{p_L,w_H,w_L} \mu_H (S(H,w_H) - V(H,w_H) - W) + \mu_L p_L (S(L,w_L) - V(L,w_L) - W) \]

subject to \((IC_H)\): \(V(H,w_H) \geq p_L V(H,w_L)\) and \((IC_L)\): \(p_L V(L,w_L) \geq V(L,w_H)\).

We are left to show that we can ignore the non-binding constraint \((IC_L)\) in this problem. Since \(S(\theta,w) - V(\theta,w) - W = \pi(\theta,w) - W\) is concave in \(w\) and \(V(\theta,w)\) is convex in \(w\) by (14), the second order condition is satisfied for \(w_L\) regardless of \(w_H\) if we ignore \((IC_L)\). Hence, we can ignore the non-binding constraint \((IC_L)\).