

Minimum Wages, Inequality, and the Informal Sector*

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Abstract

What are the effects of the minimum wage in countries with large levels of informality? This paper studies this question in the Brazilian context throughout the 2000s. Using household survey data, I show that, differently from the formal sector, informal earnings inequality did not fall alongside the rapid expansion of the minimum wage. Moreover, I provide reduced form evidence suggesting that larger minimum wages are positively related to larger shares of informal employment and larger inequality in informal earnings. Motivated by these empirical relationships, I develop a stylized model where heterogeneous firms compete for labor and select into informality to understand when increasing the minimum wage can increase overall earnings inequality and reduce welfare. I extend the model to incorporate worker heterogeneity, calibrate the quantitative framework using Brazilian data, and find that, by generating substantial amounts of informality, the observed increase in the minimum wage is responsible for a 11.5% increase in aggregate inequality. My analysis also highlights that the estimated increase in formal enforcement does little in preventing the inequality-increasing effects of the minimum wage, as well as the roles played by skill composition and skill biased technical change in understanding movements in earnings inequality and the share of informal labor.

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1 Introduction

Countries in Latin America experienced substantial decreases¹ in earnings inequality throughout the 2000s. Minimum wage policies were found to be among the most important drivers of these patterns.² However, most studies neglect the fact that agents can avoid labor legislation by establishing informal employment relationships - a predominant feature in most low income and developing countries.³ Hence, when evaluating how consequential the minimum wage is in reducing earnings inequality, it is important to incorporate the possibility that the informal sector might respond to the economic environment, altering the effectiveness of this labor market policy. What are the macroeconomic and inequality consequences of the minimum wage when the informal margin of adjustment is considered?

I provide answers to the above question with three contributions. First, using Brazilian data, I show that although formal sector inequality fell substantially between 1996 and 2012, informal sector inequality remained high, and that this is partially due to increases in the minimum wage. Second, I develop a stylized model of monopsonistic competition with informality and minimum wages to investigate under what conditions increasing the minimum wage can increase overall earnings inequality and reduce workers' welfare. Third, I build a quantitative model that incorporates the minimum wage and other mechanisms that are important in explaining observed changes in informality and inequality, and use the calibrated model to show that, all else equal, the observed increase in the minimum wage is responsible for a 11.5% increase in aggregate inequality. Lastly, I estimate that, to overcome these *unintended consequences* of the minimum wage, government authorities should have nearly doubled their efforts in enforcing formality upon Brazilian firms over the sample period.

In the first section of the paper, I use Brazilian household survey data from 1996-2012 to establish four stylized facts on informality, inequality and the minimum wage. The first stylized fact shows that informality accounts for a large share of the labor force (around 35%), and that

¹See Lustig, Lopez-Calva, and Ortiz-Juarez (2013) for Latin America; Firpo and Portela (2019) and Alvarez et al. (2018) for the case of Brazil.

²See Engbom and Moser (2021) and Haanwinckel (2020).

³See Tornarolli et al. (2014) and Medina and Schneider (2019) for the analysis in Latin America and the world as a whole, respectively.

informal workers earn lower wages, are less educated, younger and more predominantly women. The second stylized fact is that, in contrast with the sharp reduction from 0.65 to 0.33 in the variance of formal log earnings, inequality within informal sector fluctuated around 0.65. The third stylized fact dissects the movements in inequality, and shows that a strong decrease in the 90-50 percentile (p90-p50) ratio of informal earnings was compensated by an equally strong increase in the 50-10 percentile (p50-p10) ratio, explaining why informal inequality was relatively flat over the sample period. The fourth stylized fact confirms that there was a substantial increase in the real value of the minimum wage throughout this period.

I then follow the minimum wage literature (Lee, 1999, Autor, Manning, and Smith, 2016, and Engbom and Moser, 2021) to provide reduced form evidence on the relationship between the minimum wage and aggregate, formal, and informal inequality, and the informal share of labor. I find that increases in the minimum wage are associated with increases in the informal share of labor and informal inequality. Moreover, and in line with the third stylized fact above, there is a strong negative relationship between the minimum wage and the p90-p50 informal inequality, and a strong positive relationship between the minimum wage and the p50-p10. When looking at the formal sector, my findings suggest a significant negative relationship between the minimum wage and inequality, in line with the minimum wage literature. Hence, the reduced form exercises suggest important interactions between the minimum wage, the informal earnings distribution, and the informal share of labor.

To further understand these relationships, in the second part of the paper I develop a model of heterogeneous firms that compete monopsonistically in the labor market, and decide whether to operate formally or not. Firms trade off revenue losses due to government inspections when informal versus minimum wage restrictions when formal. In equilibrium, more productive firms select into formal markets, and, within formal firms, the least productive bunch at the minimum wage. In this setting, informality acts as a profitable outside option in which firms can operate if they are not productive enough to cope with the minimum wage. I use this framework to derive sharp analytical results on the effects of the minimum wage on overall inequality and worker welfare. In particular, I decompose the effects of a marginal increase in the minimum wage into the formal

and informal sector responses, and find that the net effect on overall inequality depends crucially on the elasticity of the informal sector to the minimum wage and on the relative distance between formal and informal wage distributions. Moreover, I show that when the productivity distribution is Pareto and informality levels are low, increasing the minimum wage decreases worker welfare and increases earnings inequality. These results suggest that there can be *unintended consequences of the minimum wage* on aggregate inequality under the presence of a large informal sector.

In the last two sections of the paper, I quantify the contribution of the observed increase in the minimum wage on inequality and welfare. To do so, I first extend the stylized model by considering worker heterogeneity and skill-biased technical differences in production. These quantitative features were shown to be important drivers of the informal share of labor (Haanwinckel and Soares, [Forthcoming](#)) and earnings inequality (Haanwinckel, [2020](#)), and might interact with the way in which the minimum wage affects the economy. I calibrate the model parameters using Brazilian data in 1996 and 2012, and show that the model is able to capture key moments for relative wages and earnings distributions.

In the last part of the paper, I use the calibrated model to run counterfactual exercises to assess the role of the observed increase in the minimum wage on inequality, as well as what were the economic effects of the other three mechanisms, namely labor enforcement, skill composition and skill biased technical change. I find that the increase in the minimum wage alone increases overall inequality by 11.5%, despite its effects in strongly reducing formal sector wage inequality by 19%. This comes from the fact that minimum wages generate a substantial amount of informality, increasing inequality in this sector and compensating for the inequality-reducing effects on the formal sector. Importantly, my results suggest that the estimated increase in formal enforcement does little in preventing the minimum wage to increase aggregate inequality. I estimate that an increase of at least 112% in the cost of informality is needed in order for the minimum wage by itself to reduce inequality. In line with the literature, I find that the educational improvement in the Brazilian labor force is indeed an key driver of the observed reduction in informality: the observed change in the skill composition reduces the share of informal workers by 41%. Lastly, I show that the model-implied skill biased technical change increases by 50% the share of informal

workforce and by 26% overall earnings inequality.

Related research. This paper contributes to three strands of literature. First, I relate to the body of work that studies the informal sector in developing economies, nicely summarized in Ulyssea (2020). Within these papers, I contribute to the empirical work on the informal sector (Porta and Shleifer, 2008, La Porta and Shleifer, 2014, Almeida and Carneiro, 2012, Engbom, Gonzaga, et al., 2021) by highlighting and dissecting the lack of movement in informal inequality in 2000s Brazil, and providing reduced form evidence suggesting that this is correlated with movements in the minimum wage. My paper is also related to the set of papers that approach the informal sector in models of firm heterogeneity (Ulyssea, 2010, Leal Ordóñez, 2014, Meghir, Narita, and Robin, 2015, Ulyssea, 2018, Dix-Carneiro et al., 2021). I complement this literature by developing a model of monopsonistic competition, firm heterogeneity, and minimum wages that delivers realistic wage distributions for workers of different education levels, and quantifying the effects of the minimum wage, among other factors, on aggregate, formal, and informal earnings inequality.

Second, this paper relates to the literature that studies the effects of the minimum wage in economies that are predominantly formal (Lee, 1999, Dickens and Manning, 2004, Autor, Manning, and Smith, 2016, Card et al., 2018, Harasztosi and Lindner, 2019, Dustmann et al., *Forthcoming*, Engbom and Moser, 2021, Haanwinckel, 2020, Berger, Herkenhoff, and Mongey, 2021, among others). My paper is particularly related to Engbom and Moser (2021) and Haanwinckel (2020), which study the Brazilian context using two different quantitative approaches. I complement these papers in two ways. First, I adapt the reduced-form strategy from Autor, Manning, and Smith (2016) to argue that one must be cautious when extrapolating the relationship between the minimum wage and overall inequality, as it correlates with both the share of informal workers and the distribution of informal earnings. Second, I develop a quantitative model where firms select into the informal sector to quantify the effects of the minimum wage and other mechanisms such as changes in relative supply of skills.

Third, there is a recent and related literature that studies the effects of the minimum wage in economies with a large informal sector. Jales (2018) develops a density discontinuity design to estimate, in a reduced form way, the effects of the minimum wage on the joint distribution of em-

ployment and wages in Brazil. In follow up work, Jales and Yu (2020) develop a bargaining model featuring compensating differentials and self-selection to microfound the findings in Jales (2018). Derenoncourt et al. (2021) investigates the effects of the minimum wage on racial inequality and the informal sector. Using a reduced-form approach, they show that minimum wage increases are important in explaining the fall in racial earnings gap in the 2000s, but have little effects on the share of informal labor. Haanwinckel and Soares (Forthcoming) develop a quantitative model to study the main drivers behind movements in the informal share of labor. They find that the improvement in the education composition of the Brazilian labor force was the main force behind the fall in informality, and that minimum wages helped keeping the informal share at high levels.⁴ I contribute to this literature by providing reduced-form evidence on the effects of the minimum wage on the informal sector, and by developing a quantitative model that delivers realistic wage distributions, allowing me to perform counterfactual exercises and to assess the general equilibrium effects of the minimum wage on inequality within each sector and in the aggregate.

The rest of the paper is structured as follows. Section 2 empirically motivates the relevance of the informal sector when thinking about the effects of the minimum wage. Section 3 develops a stylized model to show that the presence of informality can substantially change the effects of the minimum wage on inequality and welfare. Section 4 extends the stylized model and introduces other mechanisms that are important in generating the observed changes in inequality and informal shares. Section 5 discusses the calibration and validation. Section 6 performs the counterfactual exercises. Lastly, Section 7 concludes.

2 Empirical motivation

This section empirically motivates the importance of taking into consideration the informal sector response to the minimum wage when thinking about the latter's potential to affect the economy. I first present stylized facts on the size of informality and show that, differently from the formal

⁴In the last section of their paper, Haanwinckel and Soares (Forthcoming) discuss the implications of their model for the effect of the minimum wage on relative wages across skills. My analysis contributes and extends theirs in two ways. First, I analyze the effects of the minimum wage not only on the relative earnings across educational groups, but relative earnings across formality status. Second, and most importantly, I show that there are significant effects of the minimum wage on inequality within the formal and informal sectors.

economy, inequality levels in the informal sector did not decrease throughout the 2000s. Then, using cross-state regressions, I provide suggestive evidence that increases in the minimum wage correlate positively with informal sector inequality and the informal share of labor.

2.1 Data

The main data set in this paper is the *Pesquisa Nacional por Amostra de Domicílios* (PNAD) over the years of 1996-2012. The PNAD is a household survey with national coverage administered by the by the *Instituto Brasileiro de Geografia e Estatística* (IBGE), and it is one of the primary sources of nationally representative labor market and demographics data. The PNAD data is particularly well suited for the analysis in this paper as it contains data on both formal and informal work arrangements, which I detail below.⁵ I restrict attention to individuals highly attached to the labor force (age between 18-54), and consider only one job per worker (their main job at the reference week).⁶ I deflate all nominal variables by the CPI, and express them in terms of 2012. I follow the empirical literature on Brazil and consider monthly gross earnings as the main earnings measure. Importantly, the minimum wage in Brazil is de facto imposed at the monthly earnings level.⁷

In the PNAD households are asked whether they have a signed working permit (*Carteira de Trabalho Assinada*). When an employer signs its employee's working permit, that labor contract becomes subject to labor legislation such as the minimum wage, unemployment benefits, and others. Throughout this paper, a worker is informal if they do not have a signed working permit. Even though the share of self-employed workers is about as big as that of workers without a signed working permit, in what follows I will restrict attention to households engaging on employer-employee working relationships. To the extent that self-employment presents one additional margin in which agents can avoid having to comply with labor legislation, one can in-

⁵In figures A1 and A2, I compare the percentiles of the formal and informal earnings distribution in PNAD with those at RAIS and ECINF data sets, respectively, and confirm that PNAD is indeed a unified data set that provides a realistic picture of the earnings distribution in both sectors.

⁶Figure A3 in the Appendix shows that consistently throughout the sample period less than 5% of workers had more than one job. This pattern holds true for both formal and informal workers.

⁷Engbom and Moser (2021) show that contracted hours in the formal sector are substantially concentrated on 44 hours per week (Figure B.25) and do not respond to changes in the minimum wage (Table 3). Moreover, the PNAD survey does not have information on contracted hours, the margin over which the minimum wage actually applies to. Hence, I decide to abstract from that margin of adjustment in this paper.

interpret the results presented in this paper as a lower bound on the effects of the minimum wage on compositional changes between formal and informal sectors.

2.2 Stylized facts on the informal sector and inequality

This section establishes four stylized facts on inequality, informality, and the minimum wage. First, I contrast both formal and informal sectors in terms of demographics and earnings. Then, I show that earnings inequality in the informal sector did not fall throughout the sample period, differently from formal sector inequality. Third, I show that the reason why inequality in the informal sector did not fall is because of the increased distance between the median and the low-income workers. Lastly, I highlight the substantial increase in the real value of the minimum wage in the 2000s.

Fact 1: *The informal sector accounts for a large share of the labor force. Relative to the formal sector, the informal sector has lower wages, is less educated, younger, and has a larger share of women.*

Table 1 calculates summary statistics by formality status, for 1996 and 2012. Informality represents a large share of labor force, decreasing from 39% to 31% in the sample period. Earnings in the informal sector are on average 47% lower than formal sector earnings. There are large discrepancies in the share of workers with at least a High School diploma in the formal and informal sectors. This gap was 17 percentage points (pp) in 1996, and 23pp in 2012. Importantly, Table 1 highlights the large improvement in the supply of educated workers throughout the sample period. Lastly, the informal sector has a lower share of male workers (52% vs. 61%) and its workers tend to be younger (32 vs. 33 years old).

Table A1 in the Appendix shows that the informal sector is widespread across different industries in Brazil. The within industry share of informal workers ranges from 17% in Manufacturing to 70% in Domestic Services. Moreover, Figure A4 in the appendix performs a shift-share analysis and shows that what drives the reduction in the aggregate share of the informal sector is the reduction in informality levels within industries, rather than movements of workers across industries with different shares of informality. I rely on these statistics and the fact that there is large

Table 1: Summary statistics by formality status

	1996		2012	
	Formal	Informal	Formal	Informal
Share	60.9	39.1	69.1	30.9
Mean earnings	1,387	673	1,388	840
Share with HS	31.5	14.6	61.2	38.4
Male	63.8	55.2	58.6	50.0
Age	32.5	31.0	33.7	33.5

Notes: Earnings are deflated by CPI and expressed in 2012 values.

Sources: 1996/2012 PNAD.

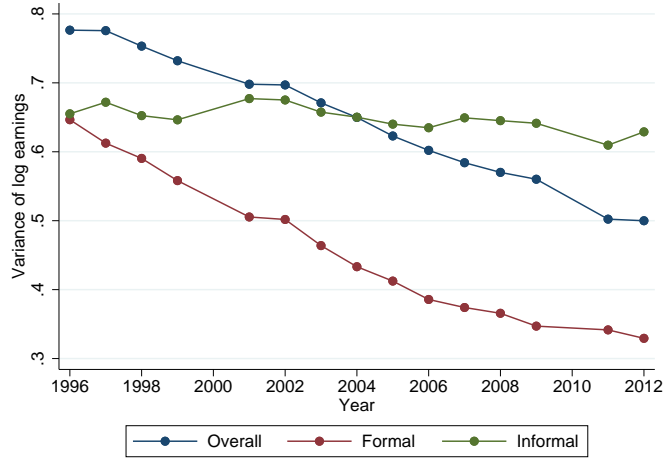
overlap in productivity distribution even within narrowly defined industries (Meghir, Narita, and Robin, 2015 and Ulyssea, 2018) to abstract from modeling industry composition explicitly. Also in the Appendix, Figure A5 shows that, relative to informal share of labor, the unemployment rate was much lower (around 7.5%) and presented lower variation throughout the sample period (9% to 6%), a result of the sample selection. This justifies another important modeling option: I abstract away from endogenous labor supply decisions throughout this paper.

Fact 2: *There was a strong reduction in overall earnings inequality throughout the 2000s. This was mostly driven by inequality reduction in the formal sector: inequality in the informal sector did not change much throughout the 2000s.*

Figure 1 plots the evolution of the variance of log earnings in the aggregate and broken down by formality status. There was a strong and steady reduction in overall (2.3% per year) and formal (4.5% per year) inequality in log earnings. At the same time, throughout the sample period, the variance of log earnings in the informal sector did not change much, fluctuating around 0.65 log points. Figure 1 also shows that throughout the 2000s there was a consistent widening in the gap between formal and informal earnings inequality.

I now argue that inequality within formal and informal sectors are important components in explaining the level and changes in overall earnings inequality. Figure A6 in the Appendix displays each component of the decomposition of the aggregate inequality into its within and

Figure 1: Variance of log earnings, 1996-2012



Sources: 1996-2012 PNAD.

between components:

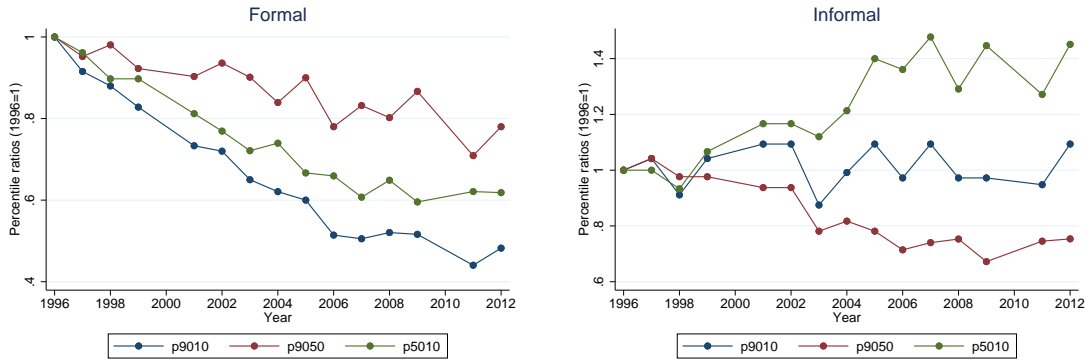
$$V_t = \underbrace{\sum_{j \in \{form, inf\}} s_{jt} V_{st}}_{\text{Within}} + \underbrace{\sum_{j \in \{form, inf\}} s_{jt} (E_{st} - E_t)^2}_{\text{Between}} \quad (1)$$

The figure shows that the weighted sum of inequality within formal and informal sectors explain over 80% of the level of aggregate earnings inequality and over 83% of the its reduction over the sample period. In what follows, I will argue both empirically and quantitatively that the minimum wage is an important driver behind these empirical patterns, in particular behind movements in inequality within formal and informal sectors as well as the informal share of labor.

Fact 3: *The reduction in inequality in the formal sector was caused by a reduction in the 50-10 percentile ratio. Inequality in the informal sector did not fall because the decrease in the p90-p50 ratio was compensated by an increase in the p50-p10 ratio. These movements are the consequence of a sharp increase in median informal earnings.*

Figure 2 decomposes, for each sector, movements in the 90-10 percentile (p90-p10) ratio into movements in the p90-p50 and p50-p10 ratios. The left plot does so for the formal sector earnings distribution. It shows that the bulk of the reduction in formal inequality was driven by the

Figure 2: Percentiles of earnings distribution, 1996-2012



Notes: Percentiles are normalized by their values in 1996.

Sources: 1996-2012 PNAD.

reduction in the p50-p10 ratio. Figure A7 in the Appendix shows that these come from a rapid increase in the real value of the 10th percentile of earnings in the formal sector.

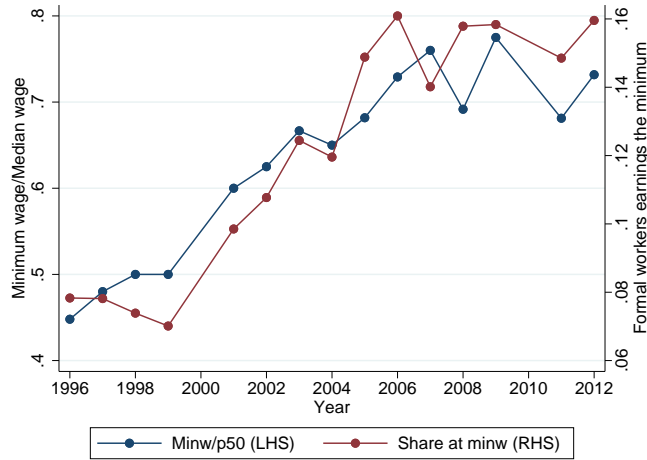
I leverage the PNAD data to understand why inequality in the informal sector has not changed over the sample period. The right plot shows that the strong decrease in inequality at the top, measured by the p90-p50 ratio, was compensated by an increase in inequality at the bottom of the earnings distribution (i.e., p50-p10). In levels, Figure A7 in the Appendix shows that the movements in the percentile ratios above were a result of a rapid increase in median earnings in the informal sector vis-a-vis stagnated 10th and 90th percentiles.

Fact 4: *The minimum wage, in real terms, increased substantially throughout the sample period.*

Figure 3 shows the evolution of the minimum wage throughout the sample period. The blue plot, associated with the left y-axis, shows that the minimum wage as a fraction of median earnings increased from 45% in 1996 to 73% in 2012. At the same time, the share of minimum wage workers in the formal sector, displayed in red and associated with the right y-axis, went up from 8% in 1996 to 16% in 2012.

What is the role of the increase in the minimum wage in explaining the movements observed both in the formal and informal sectors earnings distribution? The next section provides reduced-form evidence that minimum wages play a key role in understanding the stylized facts discussed

Figure 3: The evolution of the minimum wage, 1996-2012



Notes: Blue curve shows the fraction of median earnings that the minimum wage represents (left axis). Red curve plots the share of formal workers that receive exactly the minimum wage (right axis).

Sources: 1996-2012 PNAD.

above.

2.3 Reduced form evidence: the role of the minimum wage

This section leverages on variation at the state level over time to correlate the increases in the minimum wage with the earnings inequality in the formal sector, informal sector, and in the aggregate. There are three main takeaways: first, the results suggest that inequality in the formal sector falls with the minimum wage. Second, an increase in the minimum wage correlates positively with inequality in the informal sector and the informal share of labor. Third, and as a consequence, the reduced form relationship between the minimum wage and aggregate inequality is negative, but smaller in magnitude than the relationship with formal sector inequality. It is important to point out that the analysis in this section, in its essence, compares the evolution of outcomes in states highly exposed to the minimum wage versus states less exposed to the minimum wage. Hence, in its nature, this analysis abstracts from potential general equilibrium effects that the minimum wage might have in the economy, as these will be captured by the control variables. The general equilibrium effects of the minimum wage will be properly accounted for by the quantitative model and counterfactuals studied in the next sections.

This analysis closely follows the empirical framework and methods in Autor, Manning, and Smith (2016).⁸ I use the log-distance between the minimum wage and the median wage in the formal sector (also known as the Kaitz index) as a measure for how restrictive the minimum wage is for state s in year t :

$$\text{Kaitz}_{st} \equiv \log \left(\frac{w_t}{w_{50,F}^{st}} \right). \quad (2)$$

I correlate the minimum wage with different measures of earnings inequality (y_{st}) by regressing:

$$y_{st} = \beta_1 \cdot \text{Kaitz}_{st} + \beta_2 \cdot \text{Kaitz}_{st}^2 + \alpha(s, t) + \varepsilon_{st}, \quad (3)$$

where $\alpha(s, t)$ represents controls at the state and year level. These controls absorb state and national-level changes in the shape of the wage distribution that are not related to the minimum wage. I also experiment controlling for the unemployment rate in state s time t as a proxy for heterogeneous shocks to a state's labor market.⁹ My preferred specification follows Engbom and Moser (2021) and includes state fixed effects and state-specific quadratic time trends. It is assumed that, conditional on $\alpha(s, t)$, the error term ε_{st} is uncorrelated with the Kaitz index. The marginal coefficient on the minimum wage is estimated as: $\rho = \hat{\beta}_1 + 2\hat{\beta}_2 \overline{\text{kaitz}}$, and I evaluate it at the employment-weighted median Kaitz index.

Table 2 shows the main results of the reduced-form exercise. Each row corresponds to a different inequality measure, and each column corresponds to a specific distribution of earnings. The last row calculates the relationship between the minimum wage and the informal share of labor. The analysis in the first column shows that there is a strong, negative relationship between minimum wage and formal inequality. Importantly, despite the different data sets and sample selection, the regression estimates in Table 2 are consistent with other evidence for Brazil. Moreover, notice that the association with the minimum wage is stronger when considering the p50-p10 inequality, as low-paid workers in the formal sector tend to be those that benefit the most from an increase in the minimum wage.

⁸See Lee (1999), Haanwinckel (2020), and Engbom and Moser (2021) for papers with similar specifications.

⁹For instance, Costa, Garred, and Pessoa (2016) and Adão (2016) study the regional effects of the commodity boom in 2000s Brazil.

Table 2: Reduced form evidence on the effects of the minimum wage

Outcomes	Formal	Informal	Aggregate
Variance	-0.415***	0.085**	-0.103***
P90/P10	-1.061***	0.101*	-0.310**
P90/P50	-0.297***	-0.433***	-0.567***
P50/P10	-0.764***	0.534***	0.257***
Informal share:		0.052**	

Notes: Each cell represents a separate regression. Each cell reports the marginal coefficient on the minimum wage ($\rho = \hat{\beta}_1 + 2\hat{\beta}_2\overline{kaitz}$), where the β -coefficients are obtained by regressing (3). All specifications control for state fixed effects and state-specific quadratic time trends. Marginal coefficients are evaluated at median wage. All regressions are employment-weighted and have 405 observations (27 states by 15 years). Standard errors are clustered at the state level.

Sources: 1996/2012 PNAD.

The second column estimates the same set of regressions, but focuses on the informal earnings distribution. The first row suggests that the minimum wage correlates positively with inequality in the informal sector. Importantly, and in line with Figure 2, rows three and four show that the lack of action in informal inequality, when measured as the p90-p10 ratio, comes from the fact that there is a strong negative association between the minimum wage and the p90-p50 that is compensated by a strong positive correlation with the p50-p10. Moreover, the last row in Table 2 reports that increases in the minimum wage are associated with increases in the informal share of labor. Hence, either through the movement of more productive workers from formal to informal jobs (Jales, 2018), or through competition effects in the labor markets (Derenoncourt et al., 2021), the reduced form evidence suggests that there exists a non-trivial relationship between the minimum wage and the informal sector.

Lastly, the third column looks at the association between the minimum wage and aggregate distribution of earnings. This takes into consideration not only the within-sector associations discussed above, but also how the minimum wage is related to the distance between mean earnings in the formal and informal sectors. The relationship between the minimum wage and aggregate inequality is negative, but (as expected) smaller in magnitude than the relationship with formal sector inequality, due to the counteracting forces presented by the informal sector earnings distribution. It is important to reinforce that these results should not be read as the total effects of

the minimum wage on aggregate inequality, as these coefficients do not incorporate nation-wide effects of the minimum wage over the sample period (i.e., the “missing intercept” problem). The full assessment of the effects of the increase in the minimum wage is the main focus of Section 6.

I now discuss the robustness of these results. Figure A8 in the Appendix shows that the results are robust to different specifications of the control variables: controlling for unemployment rate, state-specific linear time trends, no state fixed effects as in Lee (1999), state-specific linear time trends and national quadratic time trends as in Haanwinckel (2020), among others. To address the possibility that the minimum wage in Brazil has effects up to the 50th percentile, Figure A9 shows that the results are similar in a specification that centers the analysis around the 90th percentile of formal earnings distribution instead of the median. I use two different strategies to control for the possibly mechanical endogeneity of the Kaitz index, as it might correlate with the residual term because median wages are used to construct both the independent and the dependent variables in rows 3 and 4. First, I redo the analysis with the share of formal workers at the minimum wage as the measure for how binding the minimum is in a given state-year. Second, I follow the 2SLS IV approach from Autor, Manning, and Smith (2016), where the first stage projects the Kaitz index and its square on log minimum wage, its square, and its interaction with the state’s overall median earnings throughout the sample period, thus filtering for transitory shocks on median wages. The results are similar and displayed in Table A2 and Figure A8, respectively. Figure A10 compares the estimates of the effect of the minimum wage on informal share of labor with those found at Engbom and Moser (2021), and shows that if we apply similar sample restrictions I also obtain a null relationship between the minimum wage and informal share. Lastly, Table A3 shows that the results are robust to the construction of a data-set with more observations, exploring heterogeneity at the state (27), industry (13) and year (15) level.

This section used Brazilian household survey data to argue that minimum wages have non-trivial associations with earnings inequality in both formal and informal sectors, as well as with the informal share of labor. In what follows, I first develop a stylized model to understand the mechanisms behind the interactions between the minimum wage, the informal sector, inequality, and worker welfare. Then, I extend the model to take it to the Brazilian data and quantify the role

of the minimum wage on earnings inequality throughout the 2000s.

3 Informality and the effects of the minimum wage

This section develops a stylized model to understand the effects of the minimum wage when the informal margin of adjustment is taken into account, before later extending the framework in a number of directions in the quantitative analysis. The model consists of homogeneous workers and heterogeneous monopsonists that decide to operate formally or not. The formal technology is subject to the minimum wage, whereas the informal technology is subject to government detection costs.

I show that, in general, whether increasing the minimum wage improves welfare (or reduces inequality) depends on how strong the response of the informal sector is. When firm productivity is Pareto-distributed, increasing the minimum wage improves welfare and reduces inequality when informality is absent, but *reduces welfare* and *increases inequality* in a model with informality. Hence, when the informal margin of adjustment is taken into account, there are *unintended consequences of the minimum wage*.

3.1 Labor supply

There exists a unit measure of ex-ante homogeneous households. Each agent is endowed with one unit of time, supplied inelastically as labor. Households receive wage offers and must choose, after the realization of firm-specific amenity shocks, which firm to work for (Card et al., 2018).

The indirect utility of household i working at firm j is:

$$V_i(j) = \max_{c \geq 0} \{A_i(j)c \mid c \leq w(j)\} = A_i(j)w(j), \quad (4)$$

where $A_i(j)$ is an amenity shock household i gets for working in firm j , and $w(j)$ is their wage. I assume that $A_i(j)$ is independently distributed across households and firms, and drawn from a Fréchet distribution with shape parameter η .

The probability¹⁰ that household i works for firm j is:

$$Prob_i(j) = \frac{w(j)^\eta}{\int_{j' \in \Omega} w(j')^\eta dj'} = \left[\frac{w(j)}{W} \right]^\eta = l(j), \quad (5)$$

with $W \equiv \left[\int_{j' \in \Omega} w(j')^\eta dj' \right]^{1/\eta}$ denoting the aggregate wage index and Ω denoting the exogenous set of operating firms. Law of large numbers implies that this probability equals the labor supply for firm j , $l(j)$.

The expected utility of households in this economy proportional to the aggregate wage index:

$$\mathbb{E}[U] = \Gamma\left(\frac{\eta-1}{\eta}\right) W. \quad (6)$$

where $\Gamma(\cdot)$ denotes the gamma function. Importantly, policies that increase the aggregate wage index are welfare improving. As I discuss further in the next sections, the possibility that firms operate the informal technology is a way for them to maintain monopsony power and avoid the minimum wage. Hence, informality is a phenomenon that deteriorates the welfare-improving effects of a small increase in the minimum wage under monopsony power. On the other hand, informality prevents disemployment effects of a large increase in the minimum wage (Dix-Carneiro et al., 2021 and Berger, Herkenhoff, and Mongey (2021)).

3.2 Labor demand

I assume there is an exogenous mass of firms that are heterogeneous with respect to labor productivity. The productivity distribution follows $z \sim F$ over $[z_0 > 0, \infty)$, with $f(z) > 0$ for all z , and $\lim_{b \rightarrow \infty} \int_{z_0}^b z^\eta dF(z) < \infty$. In the goods market, firms are perfectly competitive,¹¹ producing homogeneous goods that are perfect substitutes, which price is normalized to one. In the labor market, firms operate under monopsonistic competition.

Firms first choose the formality status to operate, then set wages and labor subject to the labor

¹⁰See Appendix C for the derivation of the employment probabilities and the wage index.

¹¹Appendix D generalizes the results in this section for an environment of monopolistic competition and love for varieties.

supply curve (5) and technological constraints. The key tradeoff at the firm level is whether to operate formally, subject to the minimum wage, or informally, subject to government detection costs.

By operating the informal technology, a firm with productivity z has profits:

$$\pi^{inf}(z) = \max_{\{l,w\}} \left\{ (1-\rho)zl - wl \quad \mid \quad l = \left(\frac{w}{W} \right)^\eta \right\}, \quad (7)$$

where $\rho \in (0, 1)$ represents the probability the informal firm is detected by government authorities and loses all its revenues.¹² I refer to ρ as the cost of informality. The resulting wage policy, labor demand, and profits are:

$$w^{inf}(z) = \frac{\eta}{\eta+1}(1-\rho)z, \quad l^{inf}(z) = W^{-\eta} \frac{\eta^\eta}{(\eta+1)^\eta} (1-\rho)^\eta z^\eta, \quad \pi^{inf}(z) = W^{-\eta} \frac{\eta^\eta}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} z^{\eta+1}. \quad (8)$$

The monopsonistic competition assumption implies that informal firms set wages as a mark down over the marginal product of labor, which is affected by the cost of informality, as it scales down labor productivity. The absence of fixed costs implies positive profits for all firms in the informal sector. Hence, informality acts as a profitable outside option for all firms in the economy.

The monopsonist with productivity z operating the formal technology is subject to the minimum wage \underline{w} , but not to informality costs, and has profits:

$$\pi^{form}(z) = \max_{\{l,w\}} \left\{ zl - wl \quad \mid \quad l = \left(\frac{w}{W} \right)^\eta, \quad w \geq \underline{w} \right\}. \quad (9)$$

Optimal wages, labor, and profits under the formal technology are:

$$w^{form}(z) = \max \left\{ \frac{\eta}{\eta+1} z, \underline{w} \right\}, \quad l^{form}(z) = W^{-\eta} \max \left\{ \frac{\eta}{\eta+1} z, \underline{w} \right\}^\eta, \quad (10)$$

$$\pi^{form}(z) = W^{-\eta} \max \left\{ \frac{\eta}{\eta+1} z, \underline{w} \right\}^\eta \left[z - \max \left\{ \frac{\eta}{\eta+1} z, \underline{w} \right\} \right]. \quad (11)$$

When unrestricted, formal firms set wages as a mark down over the marginal product of labor.

¹²Appendix E considers an alternative specification for the costs associated with the informal technology: it assumes that informal firms have an upper limit on their total employment. I show that the main intuitions extend naturally to this alternative specification of the production function.

However, when the productivity of the firm is sufficiently small, the minimum wage becomes binding, and wages and labor no longer vary with firm productivity. Hence, the minimum wage operates as a fixed production cost for low productivity firms.

Optimal profits are $\pi(z) = \max\{\pi^{form}(z), \pi^{inf}(z)\}$, with corresponding labor and wages derived in (8)-(11). Proposition 1 shows that the solution for the problem of the firm consists of two thresholds, $\underline{z} < \bar{z}$, in which firms with productivity below \underline{z} operate informally, firms with productivity in $[\underline{z}, \bar{z}]$ are formal and restricted by the minimum wage, and firms above \bar{z} operate formally and unrestricted by the minimum wage. Importantly, and in line with Machin, Manning, and Rahman (2003), this simple model environment is able to generate bunching of workers at the minimum wage, a realistic feature of the data.

Proposition 1. *There exists two thresholds:*

$$\underline{z}: \frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}\underline{z}^{\eta+1} - \underline{w}^\eta \underline{z} + \underline{w}^{\eta+1} = 0 \quad \text{and} \quad \bar{z} = \frac{\eta+1}{\eta} \underline{w} \quad (12)$$

such that:

1. $\underline{w} \leq \underline{z} < \bar{z}$;
2. Firms with $z < \underline{z}$ operate informally, firms with $z \in [\underline{z}, \bar{z}]$ are formal but restricted by \underline{w} , and firms with $z > \bar{z}$ are formal and unrestricted by the minimum;
3. $\frac{\partial \underline{z}}{\partial \rho} < 0$, $\frac{\partial \underline{z}}{\partial \underline{w}} > 0$, and $\frac{\partial^2 \underline{z}}{\partial \rho \partial \underline{w}} < 0$; and
4. $\frac{\partial(z/\underline{w})}{\partial \underline{w}} = 0$

Proof. See Appendix F for details. □

At the core of Proposition 1 is the idea that, when deciding their formality status, firms are trading off fixed versus marginal costs of production. When productivity is low, larger formal revenues do not compensate for the fixed costs implied by a binding minimum wage, and firms opt for coping with larger marginal costs caused by the detection probability. On the other hand,

when firms are very productive, letting go of a fraction of revenues is too costly, so firms decide to incur the fixed costs implied by the minimum wage.

The proposition also shows that the informality cutoff increases with the minimum wage and decreases with the cost of informality. Larger minimum wages mean larger fixed costs to operate the formal technology, so a smaller share of firms will be productive enough to be formal. Smaller costs of informality compensate firms for being informal, increasing the share of firms that will optimally do so.

3.3 Equilibrium and the welfare effects of the minimum wage

This section calculates aggregate labor supply and demand, defines the general equilibrium in this economy, and studies how the welfare effect of an increase in the minimum wage is affected by the presence of the informal sector. I focus on workers' welfare and assume that government revenue and firm profits are owned by absentee households not explicitly modeled.

Because labor is inelastically supplied, the aggregate labor supply curve is independent of the aggregate wage index: $L^S(W) = 1$. On the other hand, labor demand is calculated using the cutoffs in the firm problem, together with the distribution of firm productivity:

$$L^D(W) = \int_{z_0}^{\underline{z}} l^{inf}(z) f(z) dz + [F(\bar{z}) - F(\underline{z})] l^w + \int_{\bar{z}}^{\infty} l^{form}(z) f(z) dz, \quad (13)$$

with $l^w \equiv W^{-\eta} \underline{w}^\eta$ denoting the labor demand for minimum wage firms. Importantly, Equation (13) assumes that $z_0 < \underline{z}$, so there exists an informal sector. Whenever I refer to the economy *without the informal sector/without informality*, I am assuming that $z_0 = \underline{z}$ (either because \underline{w} is too low or $\rho = 1$).

The equilibrium in this economy is a wage index W that balances out aggregate labor supply and demand:

$$L^D(W) = 1 = L^S. \quad (14)$$

The existence and uniqueness of equilibrium in this environment follows from the existence and uniqueness of the informality threshold, derived in Proposition 1. Moreover, the structure of this

model implies that policies that increase aggregate labor demand increase the equilibrium wage index, and ultimately improve household welfare.

Proposition 2 analyzes the effects of a marginal increase in the minimum wage on household welfare. It decomposes these effects into two parts. The first component is the formal sector response. This component is a direct consequence of the effect of the minimum wage in reducing the monopsony power of firms: instead of setting wage as a mark down over marginal product of labor, they are obliged to set wages at the minimum wage, increasing labor demand. The second component represents the informal margin of adjustment. An increase in the minimum wage increases the informality cutoff. Hence, firms at the cutoff now become informal, resetting wages as a markdown over discounted productivity, and adjusting downwards their labor demand.

Proposition 2. Let $l^{inf}(z)$ and l^w denote labor allocation at informal and minimum-wage firms. Assume that the minimum wage (\underline{w}) is such that $\underline{w} < z_0 < \underline{z} < \bar{z}$. Then, the marginal effect of the minimum wage on labor demand (L^D) is:

$$\frac{\partial L^D}{\partial \underline{w}} = \underbrace{\underbrace{[F(\bar{z}) - F(\underline{z})]}_{\text{Firms at MW}} \underbrace{\frac{\partial l^w}{\partial \underline{w}}}_{L^D \text{ increase}}}_{\text{formal sector response (FR>0)}} - \underbrace{\underbrace{[l^w - l^{inf}(\underline{z})]}_{L^D \text{ drop (MW} \rightarrow \text{inf)}} \underbrace{f(\underline{z})}_{\text{firms at cutoff}} \underbrace{\frac{\partial \underline{z}}{\partial \underline{w}}}_{\text{cutoff response}}}_{\text{informal sector response (IR>0)}} \quad (15)$$

If $z \sim \text{Pareto}(\nu > \eta)$ then

1. Without informality, increasing minimum wage **increases** worker welfare: $\frac{\partial L^D}{\partial \underline{w}} > 0$.
2. With informality, increasing the minimum wage **reduces** worker welfare: $\frac{\partial L^D}{\partial \underline{w}} < 0$.

Proof. See Appendix F for details. □

Importantly, these two components offset each other, so the net effect of a marginal increase in the minimum wage depends on which dominates. If F is Pareto,¹³ the informal margin of adjustment is too strong, as there are a lot of firms concentrated around the informality cutoff,

¹³The assumption of firm productivity being Pareto-distributed is common in the strand of literature on informality with heterogeneous firms (e.g., Ulyssea, 2018 and Haanwinckel, 2020). This motivates why in propositions 2 and 3 I analyze this special case.

and increasing the minimum wage reduces workers' welfare. The proposition highlights that, under the above assumptions, increasing the minimum wage would be welfare-improving in a model without informality, so the presence of the informal sector fundamentally alters the welfare consequences of the minimum wage.

3.4 Inequality, minimum wage, and the informal sector

Proposition 3 studies what are the effects of the minimum wage on earnings inequality, and how these change when the informal margin of adjustment is taken into consideration. It shows that under the presence of the informal sector, there is scope for *unintended consequences* of the minimum wage: an increase in the minimum wage can cause overall inequality to increase.

Proposition 3. *Assume that the minimum wage (\underline{w}) is such that $\underline{w} \in (\underline{w}_0, \underline{w}_0 + \varepsilon)$, where $\underline{w}_0 : z(\underline{w}_0) = z_0$ and small ε . Then, the marginal effect of the minimum wage on the variance of log earnings (V) is:*

$$\frac{\partial V}{\partial \underline{w}} = \underbrace{\frac{\partial V^{form}}{\partial \underline{w}}}_{\text{formal sector response (FR)}} + \underbrace{\frac{\partial L^{inf}}{\partial \underline{w}}}_{\text{workers become informal}} \left[\underbrace{\left(E^{inf} - E^{form} \right)^2}_{\text{wage differential}} + \underbrace{V^{inf} - V^{form}}_{\text{pre } \uparrow \underline{w} \text{ variances}} \right] \quad (16)$$

informal sector response (IR)

If $z \sim \text{Pareto}(v > \eta)$,

1. Without informality, increasing the minimum wage **reduces** inequality: $\frac{\partial V}{\partial \underline{w}} = \frac{\partial V^{form}}{\partial \underline{w}} < 0$.
2. With informality, there are **unintended consequences** of the minimum wage: $\frac{\partial V}{\partial \underline{w}} > 0$.

Proof. See Appendix F for details. □

I first discuss the results in Proposition 3 for a general productivity distribution. When informality levels are low, the marginal effect of the minimum wage on the variance of log earnings can be decomposed into two parts. The first component represents the effects of minimum wages on the formal sector earnings inequality. This term is typically negative, and is the object of study

of the aforementioned minimum wage literature. The second component corresponds to the informal margin of adjustment. It depends upon the responsiveness of the informal sector to the increase in the minimum wage, as well as how “spread apart” the earnings distribution in both sectors are. Hence, the net effect of the minimum wage on aggregate inequality is ambiguous, depending on which component dominates.

Importantly, when firm productivity is Pareto-distributed, the net effect of a marginal increase in the minimum wage on inequality is positive: there are unintended consequences of the minimum wage. In a model without informality, inequality goes down as the minimum wage increases. However, this is only part of the story. Workers that become informal spread out the earnings distribution, raising inequality levels above and beyond the inequality-reducing effects the minimum wage has in the formal sector.

In this section, I developed a theoretical model of firm competition in the labor market, and derived sharp analytical results on the effects of the minimum wage on inequality and welfare when firms can avoid it by operating informally. I argued that a marginal increase in the minimum wage might actually *increase* earnings inequality when the informal margin of adjustment is too strong. However, as pointed out in Section 2, the increase in the Brazilian minimum wage was far from marginal. Moreover, in this simplistic framework, the increase in the minimum wage can only increase the share of informal labor, a feature that goes against what is observed in the data (see Table 1). In what follows, I develop and calibrate a quantitative model with features that accommodate the observed movements in informality and inequality, and quantify the effects of the observed change in the minimum wage on the economy.

4 Quantitative extension

This brief section describes the extended model, used to quantify the general equilibrium effects of the observed increase in the minimum wage. I add skill heterogeneity on the household side to incorporate changes in relative supply of skills and technological skill bias as a potential drivers of earnings inequality, and as a way to rationalize the decrease in the informal share of labor observed throughout the sample period. Importantly, both features considered are consistent

with empirical evidence for Brazil over the sample period.

4.1 Labor supply

Skills are exogenous, and there is a fraction N_h of workers of each skill $h = 1, \dots, H$, with $\sum_h N_h = 1$. Labor is supplied inelastically, and the value of worker i of skill h working at firm j is:

$$V_{ih}(j) = A_i(j) \cdot (1 + \varsigma_h(j))w(j), \quad (17)$$

where $\varsigma_h(j) = 0$ for all h if firm j is informal and $\varsigma_h(j) = \varsigma_h$ if formal. This formulation allows for a wedge between the nominal and the perceived value of a worker's wage, which may occur due to the value accrued to different labor legislation (e.g., vacation stipend, unemployment and retirement benefits, among others). In other words, the quantitative model allows for the possibility that one dollar of formal earnings to be worth more (or less) than one dollar of informal earnings.

The structure of amenity shocks is the same, so the labor supply curve firm j faces in the market for skill h is:

$$l_h(j) = N_h \left[\frac{(1 + \varsigma_h(j))w_h(j)}{W_h} \right]^\eta, \quad W_h = \left[\int_{j \in \Omega} [(1 + \varsigma_h(j))w_h(j)]^\eta dj \right]^{\frac{1}{\eta}} \quad (18)$$

with W_h being the wage index for skill h . The welfare of a consumer with skill h is proportional to the wage index for its skill group, W_h .

4.2 Labor demand

Heterogeneous firms aggregate labor from different skills in a CES fashion to produce a single, homogeneous good, sold under perfect competition. Importantly, the minimum wage restriction applies to workers of all skill levels.

An informal firm with productivity z has profits:

$$\pi^{inf}(z) = \max_{\{l_h(z), w_h(z)\}_h} \left\{ (1 - \rho)z\ell(z) - \sum_h w_h(z)l_h(z) \right\} \quad (19)$$

$$\text{s.t. } \ell(z) = \left[\sum_h \xi_h(z) l_h(z)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad l_h(z) = N_h \left[\frac{w_h(z)}{W_h} \right]^\eta \quad \forall h = 1, \dots, H \quad (20)$$

where ε is the elasticity of substitution across skills, and $\xi_h(z)$ is the demand shifter for skill h . Importantly, the demand shifters capture the skill bias in the production function.

Proposition 4 summarizes the solution of the informal firm's problem. Firms set wages as an adjusted markdown over marginal productivity. The adjustment term takes into consideration the demand coefficient in the production function, $\xi_h(z)$, the supply of a given skill, N_h , and the relative cost of that skill to the firm, $W_h/\mathbb{W}(z)$.

Proposition 4. *Informal profits are:*

$$\pi^{inf}(z) = \mathbb{W}(z)^{-\eta} \frac{\eta^\eta}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} z^{\eta+1} \quad (21)$$

and labor and wages for each skill $h = 1, \dots, H$:

$$w_h^{inf}(z) = [\xi_h(z)^\varepsilon / N_h]^{\frac{1}{\eta+\varepsilon}} [W_h/\mathbb{W}(z)]^{\frac{\eta}{\eta+\varepsilon}} \frac{\eta}{\eta+1} (1-\rho)z, \quad l_h^{inf}(z) = N_h \left[\frac{w_h^{inf}(z)}{W_h} \right]^\eta \quad (22)$$

where $\mathbb{W}(z)$ denotes the cost index a firm with productivity z faces:

$$\mathbb{W}(z) \equiv \left[\sum_h \xi_h(z)^{\frac{\varepsilon}{\eta+\varepsilon}(1+\eta)} \left(W_h / N_h^{1/\eta} \right)^{\frac{\eta}{\eta+\varepsilon}(1-\varepsilon)} \right]^{\frac{\eta+\varepsilon}{\eta} \frac{1}{1-\varepsilon}} \quad (23)$$

Proof. See Appendix F for details. □

A firm with productivity z in the formal sector has profits:

$$\pi^{form}(z) = \max_{\{l_h(z), w_h(z)\}_h} \left\{ z\ell - (1+\tau) \sum_h w_h(z) l_h(z) \right\} \quad (24)$$

$$\text{s.t. } \ell = \left[\sum_h \xi_h(z) l_h(z)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad l_h(z) = N_h \left[\frac{(1+\zeta_h)w_h(z)}{W_h} \right]^\eta, \quad w_h(z) \geq \underline{w} \quad \forall h = 1, \dots, H \quad (25)$$

where τ is a payroll tax rate. Similarly to ζ_h , the payroll tax rate represents a disconnect between

the nominal and the real wage for a firm which exists due to the valuation of different labor legislation. Proposition 5 shows that the above problem has a unique solution in which wages are either at the minimum wage or they reflect a mark down over the marginal product of labor. It also delivers an efficient algorithm to find the solution for the problem of the formal firm when constrained by the minimum wage.

Proposition 5. *Conditional on the productivity z , there exists a unique solution to the problem of the formal firm. In this solution, wages are either constrained at the minimum, or reflect a mark-down over the marginal product of labor.*

Proof. See Appendix F for details. □

It is a stylized fact in the informality literature that there is substantial overlap in the productivity distributions of firms in the formal and informal sectors, even within narrowly defined sectors.¹⁴ This overlap naturally translates into an overlap in the wage distributions of formal and informal workers. To capture this empirical pattern, I follow Ulyssea (2018) and consider a two-step entry procedure for the problem of the firm.¹⁵ I assume that the productivity of a firm has two components: $z = \nu\theta$, and that firms must decide their formality status with full information about ν but before θ realizes. Hence, a firm with productivity ν operates the formal technology if and only if:

$$V^{form}(\nu) \equiv \int_{\theta \in \Theta} \pi^{form}(\nu\theta) dF_{\theta}(\theta) \geq \int_{\theta \in \Theta} \pi^{inf}(\nu\theta) dF_{\theta}(\theta) \equiv V^{inf}(\nu) \quad (26)$$

Importantly, in this two-step decision process there is exit of formal firms that are not productive enough (low draw of θ) to operate with positive profits.

4.3 Equilibrium

Profits and government revenues are rebated to households that consume the final good but do not participate in production. An equilibrium is a set of wage indices W_h for all $h = 1, \dots, H$ where

¹⁴See Meghir, Narita, and Robin (2015) and Ulyssea (2018).

¹⁵An alternative way to allow for the overlap in the productivity distribution is to assume a Melitz (2003)-setup where firms are heterogeneous in their productivity and in their fixed entry costs.

aggregate labor demand equals supply:

$$\int_0^\infty l_h(z) dF(z) = N_h \Leftrightarrow W_h = \left[\int_0^\infty [(1 + \varsigma_h(z)) w_h(z)]^\eta dF(z) \right]^{\frac{1}{\eta}}, \quad \forall h = 1, \dots, H \quad (27)$$

where the if and only if comes from substituting firm-level labor supply curves. In Appendix G I calculate the market clearing condition in the goods markets.

Discussion. There are four main drivers of earnings inequality in the extended model, which in turn will be analyzed in the counterfactual Section. The first two, the minimum wage \underline{w} and the cost of informality ρ were already present in the previous section. The next two are related to the skill heterogeneity introduced. First, changes in the relative supply of different skills are incorporated through changes in N_h . Second, changes in the relative demand for different skills (i.e., changes in the skill bias of the technology) are captured via changes in $\xi_h(z)$. Next section brings the quantitative model to the data, calibrating the key parameters and discussing the external validation of the procedure.

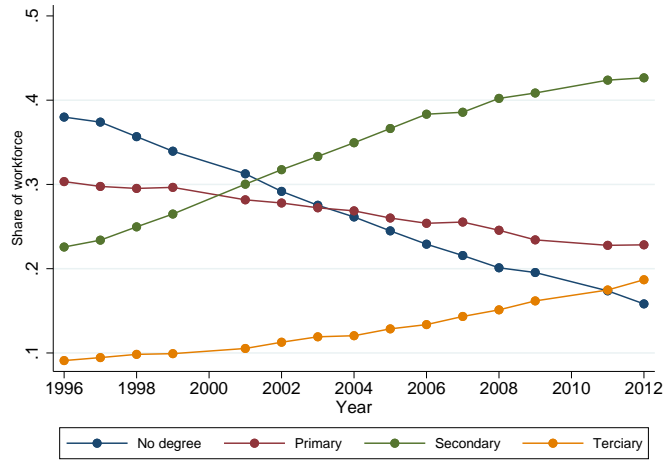
5 Calibration and validation

This section uses Brazilian data to calibrate and validate the quantitative model. I do so for 1996 and 2012, the first and last periods analyzed in Section 2. I first discuss the calibration procedure, and show that it delivers parameter values that are in line with central values in the existing empirical literature. I then perform external validation checks on model's predictions by comparing moments not targeted in the data.

5.1 Labor supply

I map workers' skills to education in the data. I construct four relatively standard education groups ($H = 4$), depending on whether a worker does not have a degree (4 years of education or less), has a primary degree (5-8 years of education), has a secondary degree (9-11 years of education), or has a tertiary degree (over 12 years of education). Figure 4 plots the share of workers

Figure 4: Skill composition of Brazilian labor force, 1996-2012



Notes: Share of labor force that belongs to each education group.

Sources: 1996-2012 PNAD.

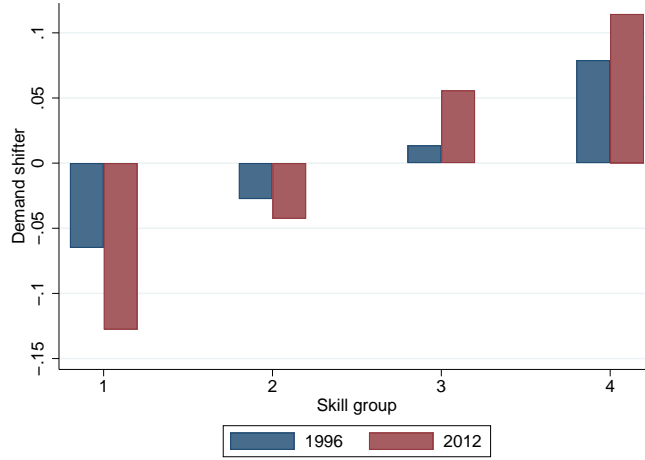
of each skill between 1996-2012, which maps directly into N_h . There was substantial increase in levels of educational attainment in the Brazilian labor force, with the share of individuals with no degree (N_1) falling from 38% in 1996 to 16% in 2012.

The definition of skills above captures substantial heterogeneity in the earnings distribution. For example, Figure A11 in the Appendix plots the distribution of log earnings relative to the minimum wage for 1996 and 2012 across different skill groups. Even though there is substantial overlapping, workers with tertiary degrees earn on average 4 times more than non-degree workers. Importantly, Figure A12 shows that the differences in mean earnings of higher educated workers relative to non-degree workers decreased substantially in the 2000s, a pattern that will be captured by the parametrization of the model.

I calibrate the elasticity of the firm-level labor supply curve to match the unconditional formal-informal mean wage gap. I obtain $\eta = 4.52$ in 1996 and $\eta = 4.22$ in 2012. The identification of this parameter comes from the fact when η is small, firm-level wages are less sensitive to firm productivity,¹⁶ attenuating the impact of formal-informal firm selection on workers' equilibrium wages. Importantly, these values are relatively constant over time, and are in the same range

¹⁶In the stylized model, for example, wages were set as $w = \frac{\eta}{\eta+1}z$ so the elasticity of wages with respect to z depends directly on how large η is.

Figure 5: Calibrated demand shifter parameters (ϕ_h)



Sources: 1996-2012 PNAD and model simulations.

of values estimated by the labor literature (e.g., Card et al. (2018) and Lamadon, Mogstad, and Setzler (2019)).

5.2 Labor demand

I incorporate skill bias in the technology by assuming that more (less) productive firms are more intensive in skilled (unskilled) labor. In particular, I assume a structure for the demand shifters similar to that in Burstein and Vogel (2017):

$$\xi_h(z) = \frac{z^{\phi_h}}{\sum_j z^{\phi_j}}, \quad \sum_h \phi_h = 0. \quad (28)$$

When $\phi_h > 0$, more productive firms are more intensive in skill h . To calibrate these parameters, I target the relative wages of different skills relative to workers with no degree. Figure 5 shows the values obtained for 1996 and 2012. The fact that the demand coefficients for skill groups 3 and 4 increase over time suggests that Brazil experienced skill-biased technical change throughout the 2000s, in line with the results in Haanwinckel (2020) and others. In other words, the calibration results suggest that more (less) productive firms became more specialized in qualified (unqualified) labor over time.

I set the elasticity of substitution across skills to $\varepsilon = 1.875$. This value lies in the range of elasticities of substitution between workers of different education levels Fernández and Messina (2018) estimates using data from Latin American countries (from 1.16 to 2.51).¹⁷ Lastly, I assume that the first productivity component, ν , is drawn from a Log-Normal distribution where the underlying Normal has mean zero and standard deviation σ . The second component of productivity, θ , is drawn from a Pareto distribution with shape parameter κ . This delivers a Pareto-LogNormal distribution of firm productivity, first introduced by Colombi (1990) and used further in the literature by Rothschild and Scheuer (2016) and Ulyssea (2018). I calibrate these parameters to match the variance of log earnings in the formal and informal sectors, resulting in $\sigma = 1.01, 1.29$ and $\kappa = 6.02, 6.33$ for 1996 and 2012, respectively. Between 1996 and 2012 there was an increase in the dispersion of the base of the productivity distribution, as σ went up, and a decrease in the tail, as κ increased.

5.3 Government

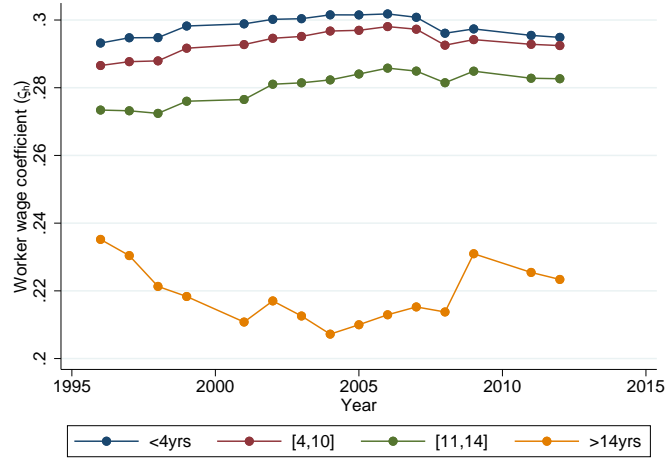
The minimum wage and the informality cost are internally estimated. I target the share of formal workers binding at the minimum wage and the overall size of the informal sector. The calibration process implies $\underline{w} = 4.04, 8.87$ and $\rho = 0.26, 0.32$ for 1996 and 2012, respectively. The model-implied 120% increase in the minimum wage is in line with the observed 106% increase in the real minimum wage. Moreover, the calibrated model delivers an increase of 23% in the informality cost. This is consistent with observed efforts by the Brazilian government to deter the growth in the informal sector and increase the enforcement of labor regulation (Corseuil, Almeida, and Carneiro, 2012).

The PNAD reports information on gross wages, and these are also the notion of wages in the model, over which the minimum wage operates. However, because of labor legislation, there is a disconnect between the perceived and the nominal value of formal wages, for both firms (τ) and workers (c_h). I now discuss how I take this into account in the quantitative exercise.

Recall that I assume that the formal firm has a total labor cost of $1 + \tau$ times its gross wage bill.

¹⁷It is also in line with values found for the US literature (Katz and Murphy, 1992).

Figure 6: Workers' valuation of wages (ζ_h), 1996-2012



Notes: Valuation of gross wages, estimated from labor legislation using the methodology in Souza et al. (2012).

Sources: 1996-2012 PNAD and Brazilian labor legislation.

This takes into consideration the fact that a firm must pay vacation stipends, social security contributions, severance payments, among other transfers, to its workers. Table B1 in the Appendix adapts the methodology from Souza et al. (2012) and estimates that, over the sample period of 1996-2012, $\tau = 71.4\%$. I assume this value to be constant over time once there was little changes in these transfers and contributions over the sample period.

Similarly, recall that each BRL a formal worker of skill h receives has a value of $1 + \zeta_h$. Appendix B details the labor legislation behind the estimation of ζ_h , which also follows the methodology in Souza et al. (2012), and are displayed in Figure 6. The figure highlights two important findings. First, the fact that $\zeta_h > 0$ for all h implies that the benefits accrued from vacation stipends, unemployment benefits, and others, more than compensates for the costs associated with income taxation and mandatory social security contributions. Second, the gap between the nominal and the real value of formal wages is 30% for no-degree workers and 22% for workers with tertiary education, this being a consequence of progressive taxation both in social security and income tax.

To summarize, Table 3 displays the parameters of the model, detailing the targets for those calibrated inside the model and sources used for those calibrated outside of the model. In terms of identification, Figure A14 in the Appendix suggests that each individual parameter plays an

Table 3: Parameters of the model

	Description	1996	2012	Target/Source
<i>1. Labor supply:</i>				
H	# of skills	4		Education groups
N_h	Skill supply	Figure 4		PNAD
η	Labor supply elast.	4.52	4.22	Formal wage premium
<i>2. Labor demand:</i>				
σ	z distribution	1.01	1.29	$V^{inf}(\log w)$
κ	z distribution	6.02	6.33	$V^{form}(\log w)$
ϕ_h	Demand shifters	Figure 5		Relative wages
ε	EoS across skills	1.875		Fernández and Messina (2018)
<i>3. Government:</i>				
\underline{w}	Min. wage	4.04	8.87	Share at min. wage
ρ	Inf. cost	0.26	0.32	Informal share
ζ_h	Earnings tax	Figure 6		PNAD and legislation
τ	Payroll tax	71.4%		Legislation

Notes: The internal calibration procedure searches for $\Theta^* = \{\phi_1^*, \phi_2^*, \phi_3^*, \sigma^*, \kappa^*, \eta^*, \rho^*, w^*\}$ that minimizes the mean absolute percentage distance between data and model moments: $\Theta^* = \operatorname{argmin}_{\Theta} \sum_{i=1}^8 |m_i(\Theta)/\hat{m}_i - 1|$, where \hat{m}_i are the targeted moments and $m_i(\Theta)$ are their model counterparts. To find the global minimum, I first evaluate the objective function at different initial points, find the resulting local minima starting at each initial point, then compare the values across minima. I used the 1996 and 2012 values for N_h and ζ_h , displayed as the initial and end points in figures 4 and 6, respectively.

Sources: 1996-2012 PNAD and model simulations.

important role in minimizing the distance between model and data moments.¹⁸

5.4 Discussion and external validation

This section discusses how well the model fits the data along targeted and untargeted moments. Table 4 compares model and data moments on relative wages across skills/sectors, earnings inequality, minimum wage bindingness, and the informal sector. The calibrated model is able to replicate targeted moments well, with the exception of inequality in the informal sector. There is an inherent tension in the model between getting the correct inequality within sectors and the right size of the informal sector: in 1996, in order to generate similar levels of inequality in the formal and informal sector, one must have a relatively low share of informal labor, whereas in 2012

¹⁸The strategy for discussing identification by evaluating how local changes in the parameter values affect the objective function was inspired by work in Adda, Dustmann, and Stevens (2017), Ulyssea (2018), and Bilal et al. (2019).

Table 4: Model moments

	1996		2012	
	Data	Model	Data	Model
<u>1. Mean earnings:</u>				
Formal/Informal	2.06	2.11	1.65	1.67
Primary/No degree	1.39	1.39	1.19	1.19
Secondary/Primary	1.46	1.49	1.21	1.21
Tertiary/Secondary	2.49	2.41	2.15	2.15
<u>2. Variance of log-earnings:</u>				
Overall	0.78	0.78	0.50	0.46
Formal	0.65	0.58	0.33	0.33
Informal	0.66	0.73	0.62	0.51
No degree	0.54	0.55	0.45	0.25
Primary	0.54	0.60	0.34	0.32
Secondary	0.64	0.63	0.32	0.37
Tertiary	0.91	0.69	0.64	0.40
<u>3. Minimum wage:</u>				
(Formal) Fraction at w	7.74	7.74	15.8	15.8
(Formal) $\frac{\text{Min wage}}{\text{Mean wage}}$	0.22	0.26	0.45	0.47
<u>4. Informal share:</u>				
Overall	0.39	0.39	0.31	0.31
No degree	0.52	0.39	0.49	0.29
Primary	0.37	0.40	0.36	0.32
Secondary	0.26	0.39	0.23	0.31
Tertiary	0.22	0.37	0.22	0.31

Notes: Bold values are moments targeted in the calibration procedure.

Sources: 1996-2012 PNAD and model simulations.

the opposite holds true - generating such discrepancy in inequality between sectors demands a larger share of informal labor.

The model does fairly well in replicating other untargeted moments, such as inequality measures within different skill groups and the relative log distance between the minimum wage and mean earnings. On the other hand, there is less heterogeneity in the informal share of labor within each skill group in the model than in the data.

Lastly, I show that the calibrated model generates realistic earnings distributions. I do so by

comparing data and model-generated histograms of log earnings relative to the minimum wage. Figure 7 displays histograms for the aggregate, formal, and informal distributions of earnings in 1996 (left column) and 2012 (right column). The top-most figures, which look at aggregate distributions, show that the model economy generates similar moments other than the mean and the variance. Moreover, the figure suggests that the Pareto Log-Normal assumption for the distribution of productivities, which ultimately shapes the wage distribution in the model, provides a good approximation for the lower and upper tails of earnings.

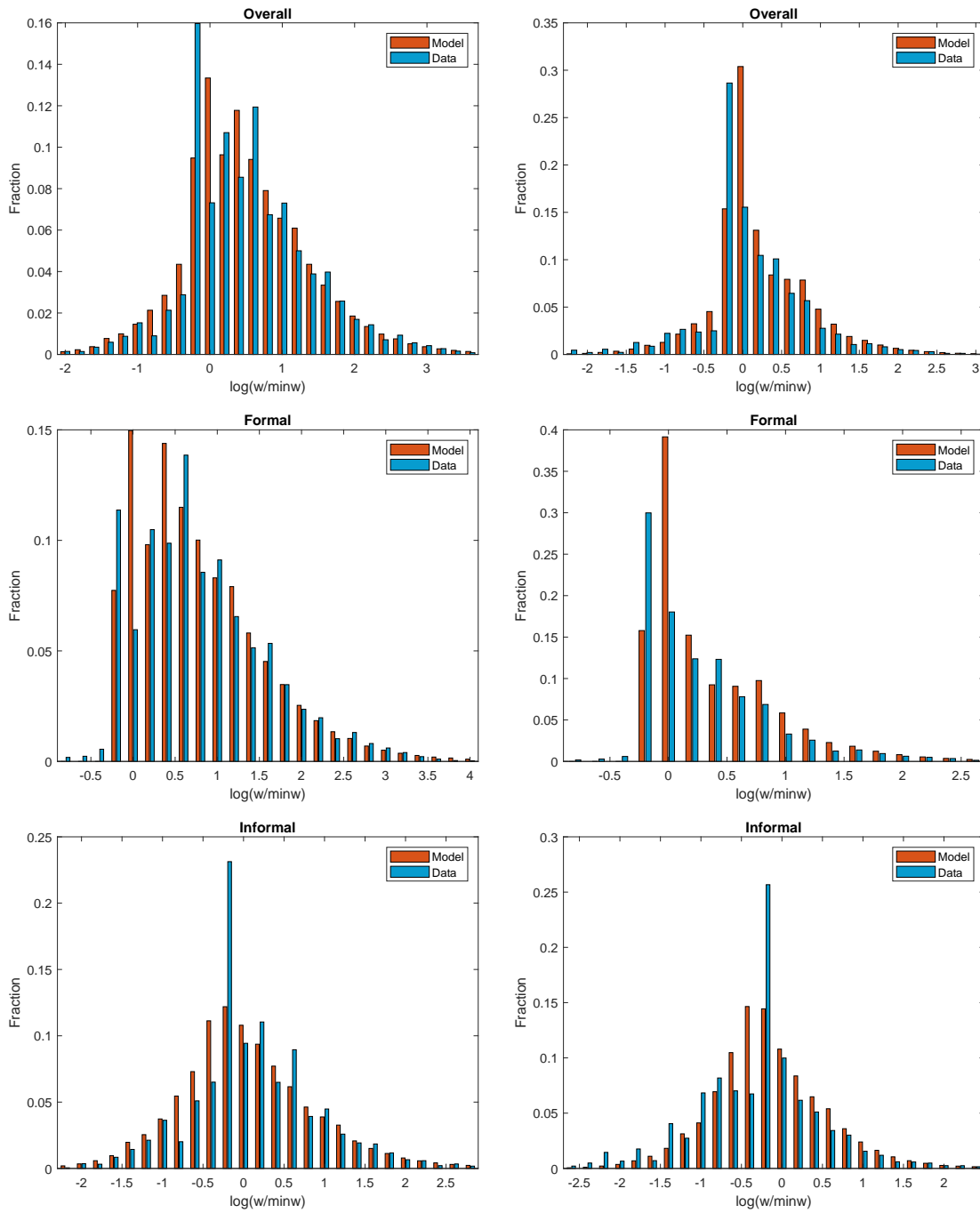
The bottom-most plots compare the earnings distribution within the formal and informal sectors in the model and data. The model is able to capture the correct bunching at the minimum wage in the formal sector, and it is again capable of generating realistic earnings distributions. The same is true for the informal sector, although there is less bunching at the minimum wage in the model than in the data, a phenomenon highlighted in Derenoncourt et al. (2021). Lastly, Figure A13 in the Appendix analyzes the within-skill earnings distribution, and shows that, yet again, the model economy reproduces realistic earnings distributions along this dimension.

This section discussed how I parametrized the quantitative model, provided evidence suggesting the calibrated model does a good job in replicating data along targeted and untargeted dimensions, and discussed some of the caveats in this process. I now turn to evaluating the counterfactual exercise: what are the welfare and inequality effects of the changes in the minimum wage?

6 Counterfactuals

The main goal of this section is to evaluate what is the role of the minimum wage in shaping the aggregate earnings distribution and the informal share of labor. I also explore the extent to which the other mechanisms at hand, namely formal enforcement, skill composition and skill biased technical change affect inequality and informality, and might interact with the effects of the minimum wage. I do so by changing each associated parameter one at a time to its estimated 2012 value and following aggregate outcomes. The main results are displayed in Table 5. The associated wage indices in 1996 and in each counterfactual scenario, which are proportionately

Figure 7: Log earnings histogram, 1996-2012



(a) 1996

(b) 2012

Notes: Histograms of log earnings relative to the minimum wage. Widths are set to 0.2. Blue histograms are model, red are data.

Sources: 1996-2012 PNAD and model simulations.

Table 5: Counterfactual exercise

	1996	Counterfactuals				2012
		\underline{w} (min wage)	ρ (enforcement)	N_h (skill comp)	$\xi_h(z)$ (SBTC)	
Mean earnings						
prim/no deg	1.39	1.37	1.39	1.03	1.76	1.19
sec/prim	1.49	1.49	1.49	0.92	2.07	1.21
terc/sec	2.41	2.42	2.41	2.35	2.31	2.15
form/inf	2.11	1.64	2.37	2.52	1.90	1.67
V(log earnings)						
overall	0.78	0.87	0.78	0.79	0.98	0.46
formal	0.58	0.47	0.58	0.62	0.70	0.33
informal	0.73	0.88	0.72	0.67	0.94	0.51
Fraction at \underline{w}	7.74	23.4	8.33	3.66	14.0	15.8
Min/mean wage	0.26	0.38	0.26	0.21	0.24	0.47
Informal share	0.39	0.86	0.28	0.23	0.61	0.31

Notes: Each column under the Counterfactuals label changes one parameter from its 1996 calibrated level to its 2012 calibrated level, displayed in Table 3. The 1996 and 2012 columns are replicated from Table 4.

Sources: Model simulations.

related to worker welfare, are shown in Table 6.

What are the effects of the increase in the minimum wage? The first three rows of the minimum wage column show that increasing \underline{w} has little power in affecting skill earnings premia. On the other hand, when minimum wages go up, there is a substantial decrease in the formal wage premium, as it is shown in the fourth row. This is a consequence of productive formal firms becoming informal, coupled with a strong increase in the share of minimum wage workers. In line with the minimum wage literature, the increase in the minimum wage is responsible for 44% of the observed decrease in formal inequality (Engbom and Moser, 2021). However, as predicted by Proposition 3, the informal margin of adjustment, that is, the increase in informality and informal inequality, implies that aggregate earnings inequality goes up by 11.5% in response to the minimum wage increase. Lastly, because of the strong informal response and the fact that informal wages are much smaller than their formal counterparts, increasing the minimum wage is welfare improving only for workers with tertiary education (+1.3% increase in welfare).

Table 6: Welfare

Wage indices (W_h)	1996	\underline{w}	ρ	N_h	$\xi_h(z)$
No degree	0.967	0.940	0.942	1.618	0.690
Primary	1.240	1.234	1.200	1.564	1.066
Secondary	1.730	1.727	1.680	1.358	1.902
Tertiary	3.687	3.734	3.593	2.733	3.952

Notes: This table plots the equilibrium wage indices expressed in Equation 27 in the 1996 baseline economy and each counterfactual scenario.

Sources: Model simulations.

I now contribute to the analysis in Ulyssea (2018) by evaluating, in 2012, which fraction of informal labor works for: (1) firms that are informal even without the spike in the minimum wage; (2) firms that are productive enough to cope with the new minimum wage but stay informal; and (3) firms that would operate formally had not the minimum wage increased. To calculate these shares, I compare the 2012 economy to a counterfactual economy in which all changes occurred except for the increase in the minimum wage, which stayed at its 1996 level. I find that 17% of informal workers are employed in type-1 firms, 2% in type-2, and 81% in firms that would be formal in the absence of the minimum wage spike. These results suggest that potentially productive informal firms that are kept out of formality by the spike in the minimum wage are majority, so formalization policies, if implemented in companion with minimum wage adjustments, have the potential to deter the increase in informality.

What are the economic effects of the other mechanisms displayed in Table 5? The increase in informality costs are associated with a 28% decrease in the share of informal labor. On top of that, there is no change in aggregate earnings inequality. This result stems from two counteracting forces: on one hand there is a strong increase in inequality across sectors, captured by the 12.3% increase in formal earnings premium. On the other hand, there is a decrease in the informal inequality. These effects net out and there is little change in overall inequality. As the rebate of government revenues accrued from formal enforcement are not taken into consideration, the analysis of the welfare effects of the change in informal costs, displayed in the second column of Table 6 is not thoroughly analyzed for brevity.

Table 7: Formalization enforcement

	1996	Counterfactuals		
		\underline{w}	$+\Delta\rho = 24\%$	$+\Delta\rho = 112\%$
V(log earnings)				
overall	0.78	0.87	0.87	0.78
formal	0.58	0.47	0.48	0.46
informal	0.73	0.88	0.85	0.82
Fraction at \underline{w}	7.74	23.4	22.6	21.7
Informal share	39.1	86.9	74.1	19.4

Notes: This table shows the counterfactual effects of a joint change in the minimum wage and the informality cost. The 1966 and \underline{w} columns replicate the results in Table 5. The $\Delta\rho = 24\%$ column evaluates the effects of changes in both the minimum wage and the estimated increase in the informality cost. The last column analyzes the joint effects of the minimum wage and a 112% increase in informality costs relative to its 1996 estimated value.

Sources: Model simulations.

Is the estimated increase in formal enforcement enough to compensate for the unintended consequences of the minimum wage? Table 7 displays the effects of the minimum wage when analyzed on its own, as well as in conjunction with changes in the informality cost ρ . The estimated increase of 24% in the informal costs does little in preventing the minimum wage from triggering a large share of informal workforce and increasing the overall earnings inequality. However, the last column suggests that if the Brazilian government had increased its formal enforcement by 112% between 1996 and 2012, then there would be no unintended consequences of the increase in the minimum wage. These results highlight the importance of formalization policies to take place in conjunction with changes in the minimum wage to prevent them from having unintended consequences on aggregate inequality.

The improvement in the skill composition is an important driver of the reduction in the share of informal labor, reducing it by 40% (Haanwinckel and Soares, [Forthcoming](#)). Low productivity firms select themselves into the informal sector (Ulysea, 2018). However, these firms are more intensive in low-skilled workers. This factor of production, however, becomes more scarce, its wages go up, and operating the informal technology becomes relatively more expensive. As more firms become formal, there is an increase of 7% in formal inequality, which is compensated by the 8% decrease in informal inequality, leaving aggregate inequality nearly unaffected. The welfare

effects of the change in the skill composition are intuitive: factors that become relatively more scarce (no-degree and secondary education workers) see an increase in welfare vis-a-vis factors that become more abundant.

Lastly, I analyze the effects of the skill-biased technical change predicted by the calibration procedure. The first, direct observation is that the shift in demand towards high skill workers with secondary and tertiary education improves their welfare by 10% and 7.2%, respectively, and the expense of decreases of 29% and 14% in welfare for no degree and primary education workers, respectively (Table 6). The SBTC increases the informal share of labor from 39% to 61%. As low productivity firms become more intensive in the factors that are the cheapest, there is an increase in expected informal profits, inducing formal firms at the margin to switch formality status. The change in the demand coefficients increase aggregate earnings inequality by 26%. This is a result of increases in inequality within both sectors - informal sector inequality expands as a direct consequence of a wider range of firms becoming informal, whereas formal sector inequality expands as there is an increase in the distance of wages paid by unconstrained and minimum wage firms.

7 Conclusion

There is a long-standing literature agreeing on the idea that minimum wages are an important tool for tackling inequality. At the same time, middle/low income countries present substantial shares informal workforce. The fact that agents can avoid labor legislation by operating informally suggests that compositional changes between formal and informal sectors are potentially important in shaping the minimum wage effects on aggregate inequality. In this paper, I present new evidence and theory on how the informal margin of adjustment influences the extent to which the minimum wage can operate as an inequality-reducing tool. In particular, I find that in the Brazilian context the spike in the minimum wage actually increased overall inequality, leading to *unintended consequences* of the minimum wage.

I reach this important conclusion in three steps. My empirical work provided reduced-form evidence that minimum wages trigger inequality increases in the informal sector, and that this

increase offsets the inequality-reducing effects the minimum wage has in the formal sector. I then devise a theoretical model and derive sharp analytical results showing that there is scope for minimum wage increases to increase aggregate earnings inequality in general equilibrium. In the last step, I build a quantitative framework to study the role of changes in the minimum wage, formal enforcement, skill composition, and SBTC on the Brazilian economy. I show that the minimum wage increase, albeit responsible for a strong reduction in formal sector earnings inequality, is also responsible for an increase of 11.5% in aggregate earnings inequality, a result that derives from a strong informal margin of adjustment.

This work opens important avenues for further research. First, it provides a tool for addressing the discussion about federal-level minimum wages in countries where local labor markets differ substantially in informality levels. For instance, in 1996 the informal share of labor across Brazilian states varied between 27% and 74%, and these might translate to different informality responses to an increase in the federal minimum wage. Second, as alternative work arrangements take place in developed countries (e.g., Uber/Lyft drivers and Grubhub/Doordash deliverers), the question of how to properly assess the effects of the minimum wage when agents can contract outside of this labor legislation becomes of first order. Lastly, and in the spirit of Berger, Herkenhoff, and Mongey (2021), the analysis of optimal minimum wage policies with an endogenous informal sector is also outside of the scope of this paper but a fruitful direction to pursue.

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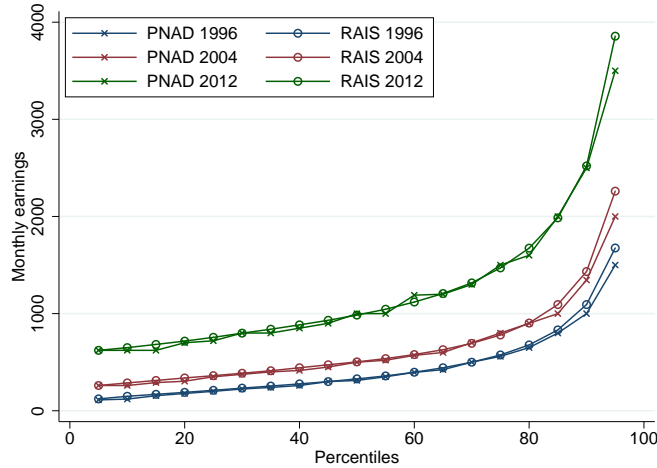
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Appendix A Additional tables and figures

This Appendix contains additional figures and tables mentioned in the main text.

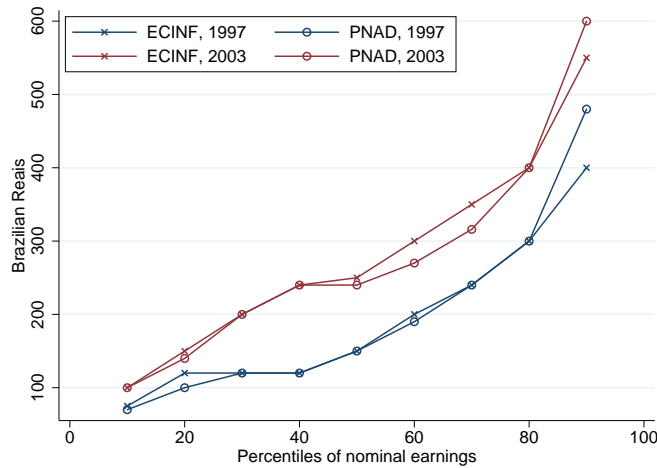
Figure A1: Comparison between RAIS and PNAD data sets, 1996-2012



Notes: Comparison between formal earnings distributions in PNAD (crosses) and RAIS (circles) across different years (colors).

Sources: 1996-2012 PNAD and RAIS.

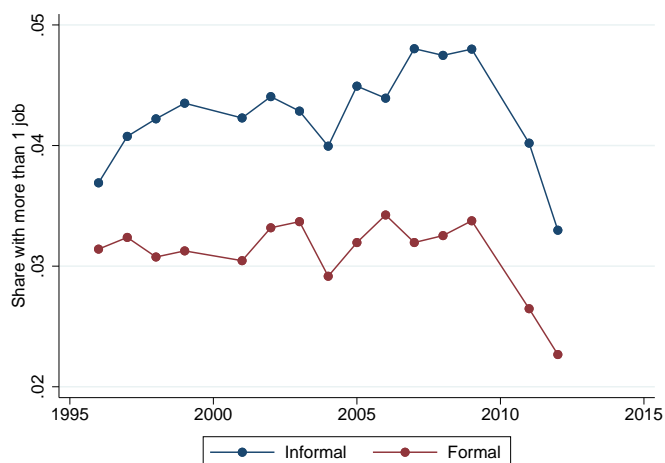
Figure A2: Comparison between ECINF and PNAD data sets, 1997 and 2003



Notes: Comparison between informal earnings distributions in PNAD (circles) and ECINF (crosses) across different years (colors).

Sources: 1997 and 2003 PNAD and ECINF.

Figure A3: Share of formal/informal workers with more than one job, 1996-2012



Notes: Share of workers in the formal and informal sectors with more than one job in the reference week.

Sources: 1997-2012 PNAD.

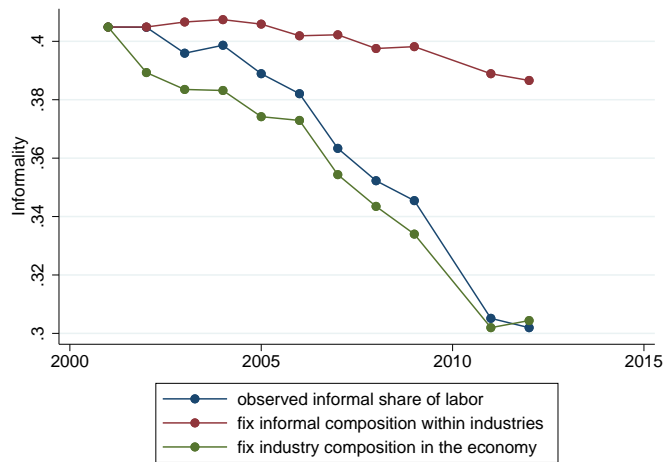
Table A1: Informal share in different industries

	Share informal	Share of total employment
Manufacturing	16.5	18.1
Other activities	16.5	9.8
Transport, storage, and communic.	20.1	5.8
Commerce and repair	24.5	18.2
Undefined	30.4	0.0
Education, health, and social serv.	32.8	9.5
Restaurant and accommodation	38.8	5.6
Construction	43.5	6.5
Other services	46.4	3.5
Public admin	55.2	3.5
Agriculture	61.6	7.8
Domestic services	69.4	11.7

Notes: Table restricts data to 2001-2012 period, as industry definitions are consistent across surveys. The second column shows the share of employment that is informal in each industry. The third column shows the size of each industry in terms of total employment. Sample weights are used.

Sources: 2001/2012 PNAD.

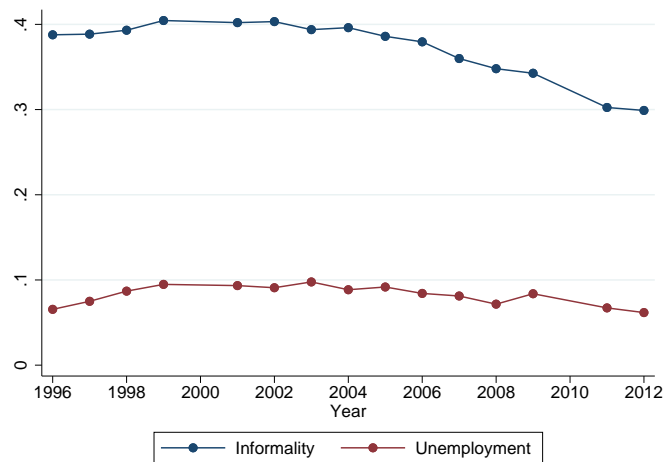
Figure A4: Shift share decomposition of informality, 1996-2012



Notes: This figure performs a shift share decomposition of the informal share of labor across different industries: $(L_t^I/L_t) = \sum_j (L_{jt}/L_t) \cdot (L_{jt}^I/L_{jt})$ where j denotes industry, t time and superscript I denotes informal employment. The blue curve shows the observed movement in informal share of labor. The red curve plots a counterfactual curve that fixes the share of informality within industries (L_{jt}^I/L_{jt}) in its initial value. The green curve plots a counterfactual curve that fixes the industry composition of the labor force (L_{jt}/L_t) in its 1996 value.

Sources: 1996-2012 PNAD.

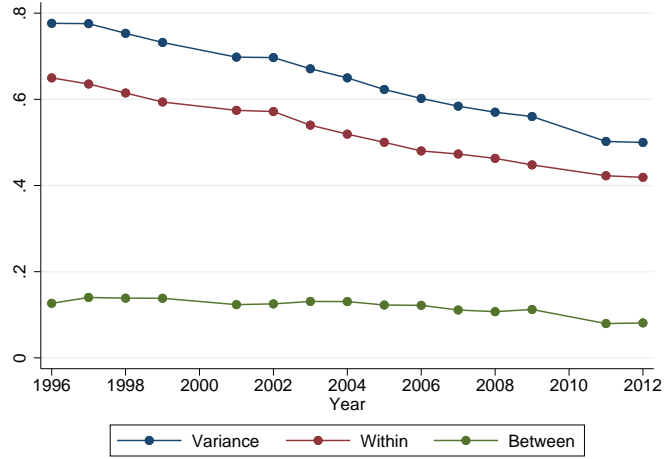
Figure A5: Informality and unemployment, 1996-2012



Notes: Blue curve shows the fraction of informal workers. Red curve plots the share of unemployed workers.

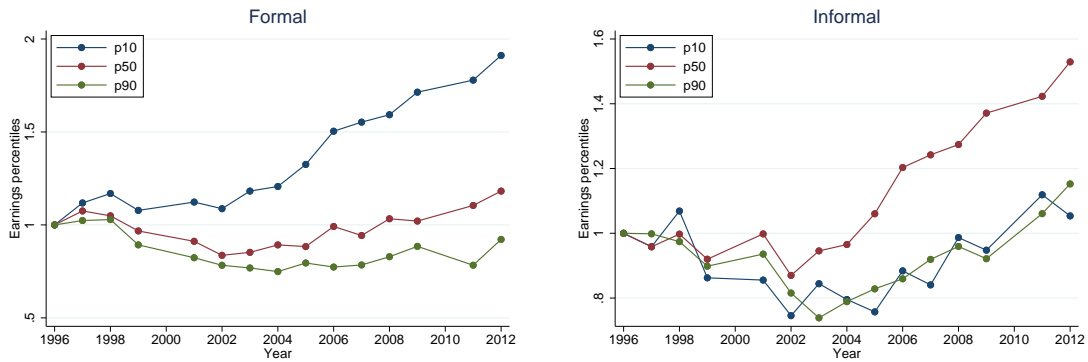
Sources: 1996-2012 PNAD.

Figure A6: Decomposition of overall variance of log earnings



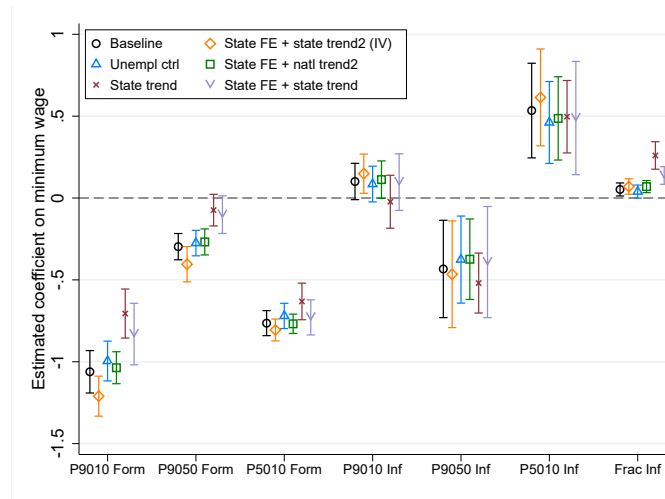
Notes: This figure decomposes overall variance in log earnings into within and between terms, following Equation (1).
Sources: 1996-2012 PNAD.

Figure A7: Percentiles of earnings distribution, 1996-2012



Notes: Percentiles are normalized by their values in 1996.
Sources: 1996-2012 PNAD.

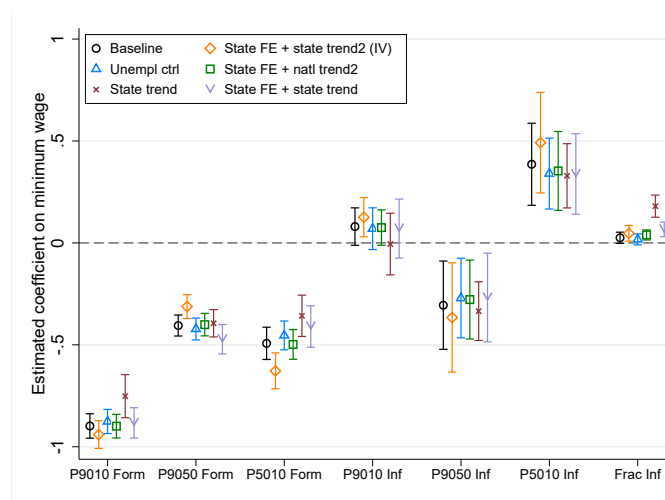
Figure A8: Marginal effect of the minimum wage (alternative specifications)



Notes: Plot shows the marginal effect of the minimum wage on different outcomes (x-axis) for different specifications (colors). “State trend” denote state-specific linear time trends, “state trend2” denote state-specific quadratic time trends, and “natl trend2” denote a national quadratic time trend.

Sources: 1996-2012 PNAD.

Figure A9: Marginal effect of the minimum wage at 90th percentile ($kaitz = \log(\underline{w}/w^{90})$)



Notes: Plot shows the marginal effect of the minimum wage on different outcomes (x-axis) for different specifications (colors). “State trend” denote state-specific linear time trends, “state trend2” denote state-specific quadratic time trends, and “natl trend2” denote a national quadratic time trend.

Sources: 1996-2012 PNAD.

Table A2: Reduced form evidence on the effects of the minimum wage (share of minw formal workers as main measure)

Outcomes	Formal	Informal	Aggregate
Variance	-0.521***	0.520***	0.302**
P90/P10	-1.647***	0.463*	1.447***
P90/P50	-0.319*	-.447*	-1.012***
P50/P10	-1.328***	0.910***	2.458***
Informal share:		-.020	

Notes: Each cell represents a separate regression. Each cell reports the marginal coefficient on the minimum wage on the regression: $y_{st} = \beta \cdot atminw_{st} + \alpha_s + \alpha_t + \varepsilon_{st}$. All regressions are employment-weighted and have 405 observations (27 states by 15 years). Standard errors are clustered at the state level.

Sources: 1996/2012 PNAD.

Figure A10: Comparison with Engbom and Moser (2021)



Notes: This figure plots the estimated marginal effect of the minimum wage on informal share of labor for different specifications. The first row (“Engbom Moser 2021”) shows the weakly positive effect of minimum wage on formal share, taken from Engbom and Moser (2021). The second row is a replication attempt of the RAIS data set with the PNAD data set. The third row includes female workers in the sample. The “Self empl” row excludes self employed workers from the sample. “Both” considers both male and female and excludes SE workers - which corresponds to the main specification in this paper. “Both+IV” uses the 2SLS strategy in Autor, Manning, and Smith (2016).

Sources: 1996-2012 PNAD.

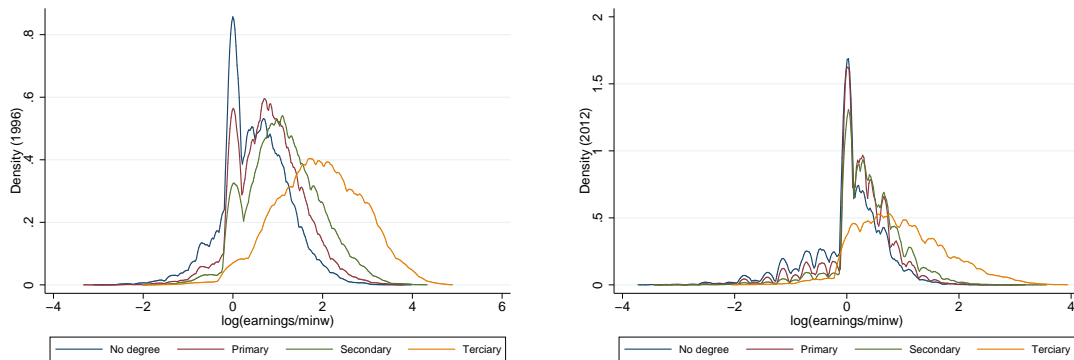
Table A3: Reduced form evidence (Industry-level analysis)

Outcomes	Formal	Informal	Aggregate
Variance	-0.337***	-0.107**	-0.222***
P90/P10	-0.948***	-0.163*	-0.400***
P90/P50	-0.227***	-0.317***	-0.400***
P50/P10	-0.720***	0.154***	-0.119***
Informal share:		0.101***	

Notes: Each cell represents a separate regression. All regressions control for state-sector fixed effects, state-sector specific quadratic time trends, and the unemployment rate. Marginal effects evaluated at median wage. All regressions are employment-weighted. Standard errors are clustered at the state level.

Sources: 1996/2012 PNAD.

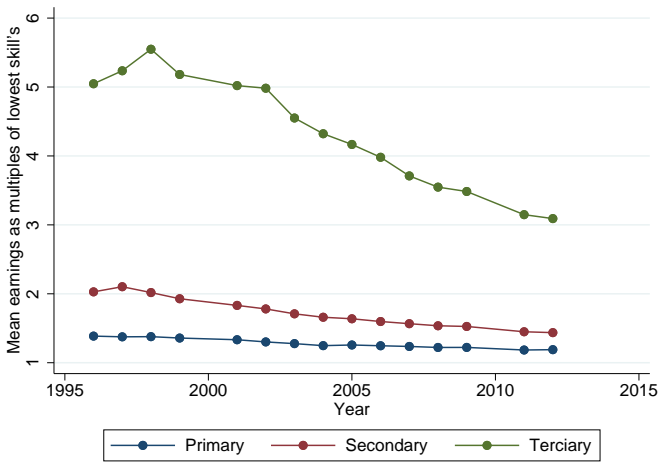
Figure A11: Earnings distribution relative to the minimum wage



Notes: Kernel density estimates for the distribution of log earnings relative to the minimum wage, by skill, for 1996 and 2012.

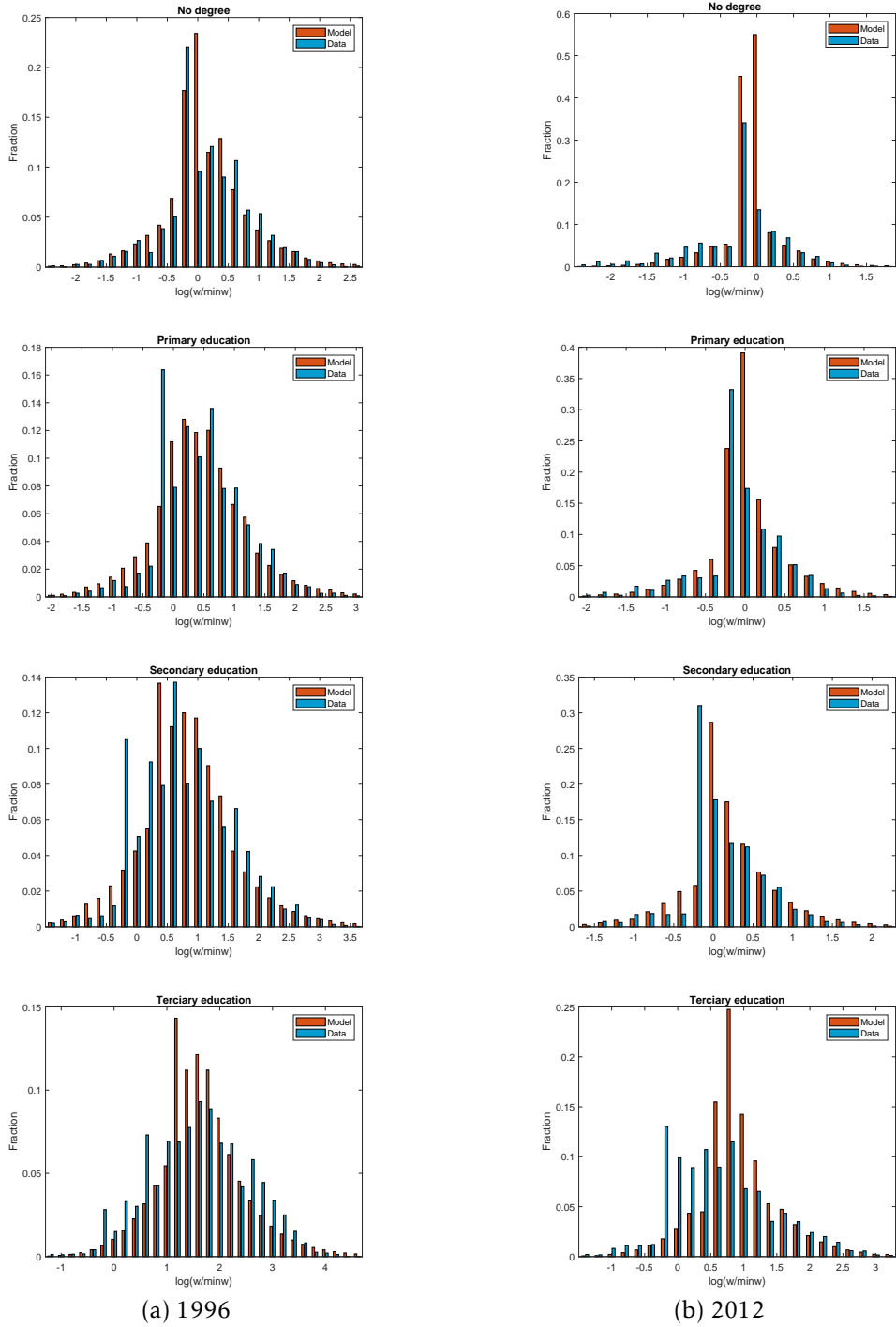
Sources: 1996, 2012 PNAD.

Figure A12: Mean earnings relative to non-degree mean earnings, 1996-2012



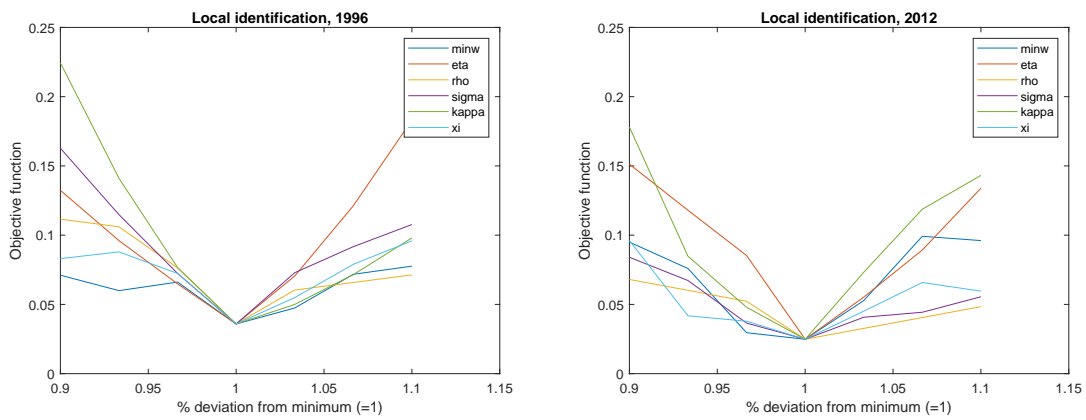
Notes:
Sources: 1996-2012 PNAD.

Figure A13: Log earnings histogram by skills, 1996-2012



Notes:
Sources:

Figure A14: Identification of calibrated parameters



Notes: Each line plots the effect of a marginal change in a parameter value on the objective function ($= \sum_{i=1}^8 |m_i(\Theta)/\hat{m}_i - 1|$). The parameters are centered around their calibrated values for 1996 and 2012. For brevity I vary all the ξ_h parameters at once, but the overall shapes in the figures do not change if I change one at a time.

Sources: Model simulations.

Appendix B Calculation of workers' and firms' wage valuations

This appendix details the methodology in Haanwinckel and Soares ([Forthcoming](#)) and Souza et al. ([2012](#)) used to calculate the wedges between nominal and real valuation of formal wages for firms and workers.

Table B1: Calculating τ

Item	Formula	Value
Nominal wage (A)		100
13th salary (A.1)	$A/12$	8.33
Vacation (A.2)	$(A/3)/12$	2.78
Advance notice	$(A+A.1+A.2)*\text{dismissed}$	3.33
Raw total (B)		114.44
FGTS Contrib (B.1)	8% of B	9.16
FGTS Fund (B.2)	$B.1*\text{duration}$	304.33
Severance payment	$B.2/2*\text{dismissed}$	4.56
INSS employer	20% of B	22.89
Other contrib	5.3% of B	6.07
Total with contrib (C)		157.12
Vacation adj	$C/11$	14.28
Total cost (D)		171.40
Payroll tax rate: τ	$D/A-1$	71.4%

Notes: Calculation of payroll tax rate τ used in the model. The above calculations were made under a dismissal probability of 3% and expected duration of employment of 33 months (Haanwinckel, [2020](#)).

Sources: Labor legislation.

[TABLE FOR WORKERS]

Appendix C Frechet calculations

This Appendix details the Frechet calculations of the labor supply curve at the firm level, as well as welfare.

Utility of household i working at firm j reads:

$$U_i(j) = A_i(j)w(j) \quad (29)$$

where the CDF and density of the iid shocks read:

$$F(A(j)) = e^{-A(j)^{-\eta}}, \quad f(A(j)) = e^{-A(j)^{-\eta}} \eta A(j)^{-\eta-1} \quad (30)$$

The share of households that optimally choose firm j is:¹⁹

$$l(j) = \int_0^1 \int_0^\infty f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)dh \quad (31)$$

$$l(j) = \int_0^1 \int_0^\infty e^{-A(j)^{-\eta}} \eta A(j)^{-\eta-1} \prod_{j' \in \Omega \setminus \{j\}} e^{-\left(\frac{w(j)A(j)}{w(j')}\right)^{-\eta}} dA(j)dh \quad (32)$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-\int_{j' \in \Omega} \left(\frac{w(j)A(j)}{w(j')}\right)^{-\eta}} dA(j)dh \quad (33)$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-(w(j)A(j))^{-\eta} \int_{j' \in \Omega} w(j')^\eta} dA(j)dh \quad (34)$$

$$l(j) = \int_0^1 \int_0^\infty \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}\right)^{-\eta}} dA(j)dh \quad (35)$$

¹⁹In the case of a discrete number of firms $j = 1, \dots, J$, the labor share allocated at firm 1 would be:

$$l(1) = \int_0^1 \int_0^\infty \int_0^{\frac{w(1)A(1)}{w(2)}} \dots \int_0^{\frac{w(1)A(1)}{w(J)}} f(A(J)) \dots f(A(1)) dA(J) \dots dA(2) dA(1)dh$$

which denotes the probability that firm 1 is chosen over all other firms $j = 2, \dots, J$ in the economy. The equation for the continuum of firms is an alternative form of expressing the same variable, calculated in Desmet, Nagy, and Rossi-Hansberg (2018).

Define $s \equiv \frac{[\int_{j' \in \Omega} w(j')^\eta]^{1/\eta}}{w(j)}$, and manipulate to find:

$$l(j) = s^{-\eta} \int_0^1 \int_0^\infty \frac{\eta}{s} \left[\frac{A(j)}{s} \right]^{-\eta-1} e^{-\left(\frac{A(j)}{s}\right)^{-\eta}} dA(j) dh \quad (36)$$

and use the fact that households are homogeneous (so integral over h is irrelevant), and that $\frac{\eta}{s} \left[\frac{A(j)}{s} \right]^{-\eta-1} e^{-\left(\frac{A(j)}{s}\right)^{-\eta}}$ is the pdf of a Frechet distribution with shape η and scale s (so it integrates to one) to find:

$$l(j) = \frac{w(j)^\eta}{\int_{j' \in \Omega} w(j')^\eta} \quad (37)$$

I now calculate what is the expected utility of a household in the model. The probability of the utility of household i being less than u conditional on firm j being its optimal choice is:

$$F_U(u) = Pr(U_i(j) \leq u | U_i(j) \geq U_i(j') \forall j' \neq j) = \frac{Pr(U_i(j) \leq u \& U_i(j) \geq U_i(j') \forall j' \neq j)}{Pr(U_i(j) \geq U_i(j') \forall j' \neq j)} \quad (38)$$

$$F_U(u) = \frac{\int_0^{\frac{u}{w(j)}} f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)}{\int_0^\infty f(A(j)) \prod_{j' \in \Omega \setminus \{j\}} F\left(\frac{w(j)A(j)}{w(j')}\right) dA(j)} \quad (39)$$

$$F_U(u) = \frac{\int_0^{\frac{u}{w(j)}} \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\left[\frac{\int_{j' \in \Omega} w(j')^\eta}{w(j)}\right]^{1/\eta}}\right)^{-\eta}} dA(j)}{\int_{j' \in \Omega} w(j')^\eta} \quad (40)$$

$$F_U(u) = \int_0^{\frac{u}{w(j)}} \frac{\int_{j' \in \Omega} w(j')^\eta}{w(j)^\eta} \eta A(j)^{-\eta-1} e^{-\left(\frac{A(j)}{\left[\frac{\int_{j' \in \Omega} w(j')^\eta}{w(j)}\right]^{1/\eta}}\right)^{-\eta}} dA(j) \quad (41)$$

$$F_U(u) = \int_0^{\frac{u}{w(j)}} \frac{\eta}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}} \left[\frac{w(j)A(j)}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}} \right]^{-\eta-1} e^{-\left(\frac{w(j)A(j)}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}\right)^{-\eta}} w(j) dA(j) \quad (42)$$

change variables and call $x = w(j)A(j)$ to find:

$$F_U(u) = \int_0^u \frac{\eta}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}} \left[\frac{x}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}} \right]^{-\eta-1} e^{-\left(\frac{x}{\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}}\right)^\eta} dx \quad (43)$$

so the optimal utility U is a Frechet random variable with shape η and scale $\left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta}$, which means that its mean is given by:

$$\mathbb{E}[U] = \Gamma\left(\frac{\eta-1}{\eta}\right) \left[\int_{j' \in \Omega} w(j')^\eta\right]^{1/\eta} = \Gamma\left(\frac{\eta-1}{\eta}\right) W \quad (44)$$

Appendix D Monopolistic competition

[Derive results with monopolistic competition in the goods market and love for variety]

Appendix E Informal size cap

[Derive results with alternative costs of informality: $l^{inf}(z) \leq \bar{l}$]

Appendix F Theory appendix

This section details the proofs for all propositions in the main text.

F.1 Proof of Proposition 1

Firms with $z \leq \underline{w}$ experience negative profits in the formal sector, hence operate the informal technology. On the other hand, when $z \geq \bar{z} \equiv \frac{\eta+1}{\eta} \underline{w}$ minimum wages do not bind, and profits in the formal sector are greater than profits in the informal sector because of the informal sector cost.

I analyze firms with $z \in (\underline{w}, \bar{z})$, where minimum wage is binding but there are positive profits in the formal sector. Define the profit gains from operating informally as:

$$\Delta(z) \equiv W^{-\eta} \left[\frac{\eta^\eta}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} z^{\eta+1} - \underline{w}^\eta z + \underline{w}^{\eta+1} \right] \quad (45)$$

It is easy to see that $\Delta(\underline{w}) > 0$ and $\Delta(\bar{z}) < 0$, as well as $\Delta'(z) < 0$ for $z \in (\underline{w}, \bar{z})$. Hence, there exists a threshold $\underline{z} \in (\underline{w}, \bar{z})$ implicitly determined by:

$$\frac{\eta^\eta}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} \underline{z}^{\eta+1} - \underline{w}^\eta \underline{z} + \underline{w}^{\eta+1} = 0 \quad (46)$$

where firms with productivity below \underline{z} operate informally, and firms with productivity above \underline{z} operate the formal technology restricted by the minimum. The ordering $\underline{w} \leq \underline{z} < \bar{z}$ follows from the above arguments.

For the third part, differentiate Equation (46) to find:

$$\frac{\partial \underline{z}}{\partial \rho} = \frac{\frac{\eta^\eta}{(\eta+1)^\eta} (1-\rho)^\eta \underline{z}^{\eta+1}}{\frac{\eta^\eta}{(\eta+1)^\eta} (1-\rho)^{\eta+1} \underline{z}^\eta - \underline{w}^\eta} < 0, \quad \frac{\partial \underline{z}}{\partial \underline{w}} = \frac{\eta \underline{w}^{\eta-1} \underline{z} - (\eta+1) \underline{w}^\eta}{\frac{\eta^\eta}{(\eta+1)^\eta} (1-\rho)^{\eta+1} \underline{z}^\eta - \underline{w}^\eta} > 0 \quad (47)$$

$$\frac{\partial^2 \underline{z}}{\partial \rho \partial \underline{w}} = \frac{\eta \underline{w}^{\eta-1} \frac{\partial \underline{z}}{\partial \rho} + \frac{\eta^\eta}{(\eta+1)^{\eta-1}} (1-\rho)^\eta \underline{z}^\eta \frac{\partial \underline{z}}{\partial \underline{w}} - \frac{\eta^{\eta+1}}{(\eta+1)^\eta} (1-\rho)^{\eta+1} \underline{z}^{\eta-1} \frac{\partial \underline{z}}{\partial \rho} \frac{\partial \underline{z}}{\partial \underline{w}}}{\frac{\eta^\eta}{(\eta+1)^\eta} (1-\rho)^{\eta+1} \underline{z}^\eta - \underline{w}^\eta} < 0 \quad (48)$$

To show the fourth part, divide the cutoff equation by $\underline{w}^{\eta+1}$ to find:

$$\frac{\eta^\eta}{(\eta+1)^{\eta+1}}(1-\rho)^{\eta+1}(\underline{z}/\underline{w})^{\eta+1} - \underline{z}/\underline{w} + 1 = 0 \quad (49)$$

hence, the ratio between the informality threshold and the minimum wage is entirely determined by η and ρ .

E.2 Proof of Proposition 2

Differentiate labor demand (13) with respect to the minimum wage:

$$\frac{\partial L^D}{\partial \underline{w}} = l^{inf}(\underline{z})f(\underline{z})\frac{\partial \underline{z}}{\partial \underline{w}} + \left[f(\bar{z})\frac{\partial \bar{z}}{\partial \underline{w}} - f(\underline{z})\frac{\partial \underline{z}}{\partial \underline{w}} \right] l^w + [F(\bar{z}) - F(\underline{z})]\frac{\partial l^w}{\partial \underline{w}} - l^{form}(\bar{z})f(\bar{z})\frac{\partial \bar{z}}{\partial \underline{w}} \quad (50)$$

but $l^{form}(\bar{z}) = l^w$, so the above becomes:

$$\frac{\partial L^D}{\partial \underline{w}} = [F(\bar{z}) - F(\underline{z})]\frac{\partial l^w}{\partial \underline{w}} - [l^w - l^{inf}(\underline{z})]f(\underline{z})\frac{\partial \underline{z}}{\partial \underline{w}} \quad (51)$$

which shows the first part of the proposition using the fact that $\bar{z} > \underline{z}$ and $l^w > l^{inf}(\underline{z})$.

To show the second part of the proposition, I will show that in the Pareto case the wage index decreases when minimum wages goes up. Given the inelastic labor supply curve, it then implies that labor demand must have decreased, which proves the result.

To simplify calculations, I study the behavior of $\tilde{W} \equiv \left(\frac{\eta+1}{\eta}W\right)^\eta$ as the minimum wage changes. When F is Pareto($z_0, \nu > \eta$), \tilde{W} takes the form:

$$\tilde{W} = (1-\rho)^\eta \frac{\nu}{\nu-\eta} \left(z_0^{-(\nu-\eta)} - \underline{z}^{-(\nu-\eta)} \right) + (\underline{z}^{-\nu} - \bar{z}^{-\nu})\bar{z}^\eta + \frac{\nu}{\nu-\eta} \bar{z}^{-(\nu-\eta)} \quad (52)$$

which can be rearranged as:

$$\tilde{W} = (1-\rho)^\eta \frac{\nu}{\nu-\eta} z_0^{-(\nu-\eta)} + \left\{ \frac{\eta}{\nu-\eta} + \left[1 - \frac{\nu}{\nu-\eta} (1-\rho)^\eta (\underline{z}/\bar{z})^\eta \right] (\underline{z}/\bar{z})^{-\nu} \right\} \bar{z}^{-(\nu-\eta)} \quad (53)$$

Hence, because (\underline{z}/\bar{z}) does not vary with the minimum wage (Proposition 1), and $1 - \frac{\nu}{\nu-\eta}(1 -$

$\rho)^\eta(\underline{z}/\bar{z})^\eta > 0$, increasing the minimum wage reduces the aggregate wage index.

E.3 Proof of Proposition 3

Throughout this proof I use a superscript I to denote workers in the informal sector, a superscript F to denote workers in the formal sector (both at and above the minimum wage), a superscript \underline{w} to denote minimum wage workers and FNB to denote formal workers above the minimum.

I start with the following variance decomposition:

$$V = L^I V^I + L^F V^F + L^I (\mathbb{E}^I - \mathbb{E})^2 + L^F (\mathbb{E}^F - \mathbb{E})^2 \quad (54)$$

that decomposes overall variance of log earnings into the within and between components, splitting workers between the formal and informal groups.

Notice that aggregate variance of log earnings is: $\mathbb{E} = L^I \mathbb{E}^I + L^F \mathbb{E}^F$. Hence, I have:

$$V = L^I V^I + (1 - L^I) V^F + L^I (1 - L^I) (\mathbb{E}^I - \mathbb{E}^F)^2 \quad (55)$$

which I differentiate with respect to the minimum wage, to find:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial L^I}{\partial \underline{w}} V^I + L^I \frac{\partial V^I}{\partial \underline{w}} - \frac{\partial L^I}{\partial \underline{w}} V^F + (1 - L^I) \frac{\partial V^F}{\partial \underline{w}} + \left(\frac{\partial L^I}{\partial \underline{w}} - 2L^I \frac{\partial L^I}{\partial \underline{w}} \right) (\mathbb{E}^I - \mathbb{E}^F)^2 + L^I (1 - L^I) \frac{\partial (\mathbb{E}^I - \mathbb{E}^F)^2}{\partial \underline{w}} \quad (56)$$

where $(\mathbb{E}^I - \mathbb{E}^F)$ denote the difference in mean log earnings in the formal and informal sectors.

Define \underline{w}_0 such that $\underline{z} = z_0$:

$$\frac{\eta^\eta}{(\eta + 1)^{\eta+1}} (1 - \rho)^{\eta+1} z_0^{\eta+1} - \underline{w}_0^\eta z_0 + \underline{w}_0^{\eta+1} = 0 \quad (57)$$

Importantly, at $\underline{w} \approx \underline{w}_0$, $L^I \approx 0$, implying:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial V^F}{\partial \underline{w}} + \frac{\partial L^I}{\partial \underline{w}} [(\mathbb{E}^I - \mathbb{E}^F)^2 + V^I - V^F] \quad (58)$$

and establishing the first part of the proposition.

I now turn to the Pareto case. To show part 1. of the second half of the proposition, I calculate the variance of log earnings in an economy without the informal sector. I then show that increasing the minimum wage reduces inequality necessarily.

Importantly, we are in the case where \underline{w} is such that $\underline{z} = z_0$, which implies that the minimum wage, though binding ($\bar{z} > z_0$), does not cause firm exit ($\underline{w} < z_0$). When $\underline{w} < z_0$, the share of the workforce at the minimum wage is:

$$L^{\underline{w}} = \frac{\frac{\nu-\eta}{\nu} \left[(w_0^F/\underline{w})^{-\nu} - 1 \right]}{\frac{\nu-\eta}{\nu} \left[(w_0^F/\underline{w})^{-\nu} - 1 \right] + 1}, \quad w_0^F \equiv \frac{\eta}{\eta+1} z_0, \quad (59)$$

and the share of workers above the minimum wage is $L^{FNB} = 1 - L^{\underline{w}}$. By inspection, it is easy to see that when \underline{w} increases, $L^{\underline{w}}$ goes up and L^{FNB} decreases.

The wage distribution (and corresponding density) takes the form:

$$G(w) = \begin{cases} 0 & w < \underline{w} \\ L^{\underline{w}} & w = \underline{w} \\ L^{\underline{w}} + (1 - L^{\underline{w}}) \cdot [1 - (\underline{w}/w)^{\nu-\eta}] & w > \underline{w} \end{cases}, \quad g(w)dw = \begin{cases} 0 & w < \underline{w} \\ L^{\underline{w}} & w = \underline{w} \\ (1 - L^{\underline{w}})(\nu - \eta)\underline{w}^{\nu-\eta} w^{-(\nu-\eta+1)} dw & w > \underline{w} \end{cases} \quad (60)$$

With mean of log earnings:

$$\mathbb{E}[\log w] = \int_{\underline{w}}^{\infty} \log w g(w) dw + \log \underline{w} L^{\underline{w}} = \frac{L^{FNB}}{\nu - \eta} + \log \underline{w}, \quad (61)$$

and mean of square log earnings:

$$\mathbb{E}^F[(\log w)^2] = 2 \frac{L^{FNB}}{\nu - \eta} \left[\log \underline{w} + \frac{1}{\nu - \eta} \right] + (\log \underline{w})^2 \quad (62)$$

so the variance is:

$$V(\log w) = \frac{2L^{FNB} - (L^{FNB})^2}{(\nu - \eta)^2}. \quad (63)$$

Notice that $2x - x^2$ is a concave parabola with maximum at $x = 1$. Hence, because we have $L^{FNB} \leq 1$,

$\uparrow \underline{w} \Rightarrow \downarrow L^{FNB} \Rightarrow \downarrow V$.

I now show part 2. of the second half of the proposition. It states that when the informal sector is present, the formal sector response is zero and the informal sector response is positive, hence variance of aggregate log earnings increases with the minimum wage, at the margin.

I proceed in three steps. First, I show that when the informal sector is present, the relative share of minimum wage workers within the formal sector workers does not change with the minimum wage, and this implies that the variance of log earnings in the formal sector does not change with \underline{w} ($FR = 0$). I then show that the share of informal workers strictly increase with the minimum wage, so $\frac{\partial L^I}{\partial \underline{w}} > 0$. I then prove that the last term, inside brackets, is also positive, implying that $IR > 0$ and hence $\frac{\partial V}{\partial \underline{w}} > 0$.

In a model with Pareto distribution and informality, the share of informal and minimum wage workers are, respectively:

$$L^I = \frac{(w_0^I)^{-(v-\eta)} - (w_1^I)^{-(v-\eta)}}{(w_0^I)^{-(v-\eta)} - (w_1^I)^{-(v-\eta)} + \frac{v-\eta}{v} \left[(w_1^I)^{-v} - (1-\rho)^{-v} \underline{w}^{-v} \right] \underline{w}^\eta + (1-\rho)^{-v} \underline{w}^{-(v-\eta)}} \quad (64)$$

$$L^w = \frac{\frac{v-\eta}{v} \left[(w_1^I)^{-v} - (1-\rho)^{-v} \underline{w}^{-v} \right] \underline{w}^\eta}{(w_0^I)^{-(v-\eta)} - (w_1^I)^{-(v-\eta)} + \frac{v-\eta}{v} \left[(w_1^I)^{-v} - (1-\rho)^{-v} \underline{w}^{-v} \right] \underline{w}^\eta + (1-\rho)^{-v} \underline{w}^{-(v-\eta)}} \quad (65)$$

Importantly, the relative share of formal sector workers at the minimum wage does not vary with the minimum wage:

$$\frac{L^w}{1-L^I} = \frac{\frac{v-\eta}{v} \left[(w_1^I)^{-v} - (1-\rho)^{-v} \underline{w}^{-v} \right] \underline{w}^\eta}{\frac{v-\eta}{v} \left[(w_1^I)^{-v} - (1-\rho)^{-v} \underline{w}^{-v} \right] \underline{w}^\eta + (1-\rho)^{-v} \underline{w}^{-(v-\eta)}} = \frac{\frac{v-\eta}{v} \left[\left(\frac{\eta}{\eta+1} \underline{z}/\underline{w} \right)^{-v} - 1 \right]}{\frac{v-\eta}{v} \left[\left(\frac{\eta}{\eta+1} \underline{z}/\underline{w} \right)^{-v} - 1 \right] + 1}, \quad (66)$$

a result that follows from Proposition 1, as the ratio $\underline{z}/\underline{w}$ is constant with respect to the minimum wage.

The distribution of earnings in the formal sector is:

$$G^F(w) = \begin{cases} 0 & w < \underline{w} \\ \frac{L^w}{1-L^I} & w = \underline{w} \\ \frac{L^w}{1-L^I} + \frac{1-L^w-L^I}{1-L^I} \cdot [1 - (\underline{w}/w)^{v-\eta}] & w > \underline{w} \end{cases}, \quad g^F(w)dw = \begin{cases} 0 & w < \underline{w} \\ \frac{L^w}{1-L^I} & w = \underline{w} \\ \frac{1-L^w-L^I}{1-L^I} (v-\eta) \underline{w}^{v-\eta} w^{-(v-\eta+1)} dw & w > \underline{w} \end{cases} \quad (67)$$

so mean log earnings in the formal sector is:

$$\mathbb{E}^F[\log w] = \frac{\alpha}{\nu - \eta} + \log \underline{w}, \quad \alpha \equiv \frac{1 - L^w - L^I}{1 - L^I} \quad (68)$$

and variance of log earnings in the formal sector is:

$$V^F(\log w) = \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} \quad (69)$$

which implies that $\frac{\partial V^F}{\partial \underline{w}} = 0$.

Using the formulas for mean and variance of log earnings in the formal sector, rewrite the marginal effect of \underline{w} on earnings inequality as:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial L^I}{\partial \underline{w}} \left[\left(\mathbb{E}^I - \frac{\alpha}{\nu - \eta} - \log \underline{w} \right)^2 + V^I - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} \right]. \quad (70)$$

However, the distribution of earnings in the informal sector is truncated Pareto:

$$G^I(w) = \frac{1 - (w_0^I/w)^{\nu-\eta}}{1 - (w_0^I/w_1^I)^{\nu-\eta}}, \quad w \in [w_0^I, w_1^I] \quad (71)$$

so mean and variance of log earnings in the informal sector are:

$$\mathbb{E}^I(\log w) = \frac{(w_0^I)^{-(\nu-\eta)} \log(w_0^I) - (w_1^I)^{-(\nu-\eta)} \log(w_1^I)}{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)}} + \frac{1}{\nu - \eta} \quad (72)$$

$$V^I(\log w) = \frac{1}{(\nu - \eta)^2} - \left[\frac{\log(w_1^I/w_0^I)}{(w_0^I)^{-(\nu-\eta)} - (w_1^I)^{-(\nu-\eta)}} \right]^2 (w_0^I w_1^I)^{-(\nu-\eta)} \quad (73)$$

for $w_0^I = \frac{\eta}{\eta+1}(1-\rho)z_0$ and $w_1^I = \frac{\eta}{\eta+1}(1-\rho)z$. When \underline{w} is such that $w_1^I \approx w_0^I$ the mean and variance of log earnings in the informal sector are:

$$\mathbb{E}^I(\log w) = \log(w_0^I), \quad V^I = 0 \quad (74)$$

so the marginal effect of the minimum wage on variance of log earnings is:

$$\frac{\partial V}{\partial \underline{w}} = \frac{\partial L^I}{\partial \underline{w}} \left[\left(\log \left(w_0^I / \underline{w} \right) - \frac{\alpha}{\nu - \eta} \right)^2 - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} \right]. \quad (75)$$

To show that $\frac{\partial V}{\partial \underline{w}} > 0$ I then show that $\frac{\partial L^I}{\partial \underline{w}} > 0$ and $[..] > 0$.

The informal share of workers takes the form:

$$L^I = \frac{(w_0^I / \underline{w})^{-(\nu - \eta)} - (w_1^I / \underline{w})^{-(\nu - \eta)}}{(w_0^I / \underline{w})^{-(\nu - \eta)} - (w_1^I / \underline{w})^{-(\nu - \eta)} + \frac{\nu - \eta}{\nu} \left[(w_1^I / \underline{w})^{-\nu} - (1 - \rho)^{-\nu} \right] + (1 - \rho)^{-\nu}} \quad (76)$$

and define $x \equiv (w_0^I / \underline{w})^{-(\nu - \eta)}$, $A \equiv (w_1^I / \underline{w})^{-(\nu - \eta)}$, $B \equiv \frac{\nu - \eta}{\nu} \left[(w_1^I / \underline{w})^{-\nu} - (1 - \rho)^{-\nu} \right]$, and $C \equiv (1 - \rho)^{-\nu}$, with A , B , and C being constants due to Proposition 1, to write:

$$L^I = \frac{x - A}{x - A + B + C} \quad (77)$$

and notice that because x increases with \underline{w} , I have $\frac{\partial L^I}{\partial \underline{w}} > 0$.

Lastly, I show that:

$$\left(\log \left(w_0^I / \underline{w} \right) - \frac{\alpha}{\nu - \eta} \right)^2 - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} > 0 \quad (78)$$

Open the quadratic term, invert w_0^I and \underline{w}_0 , and put $\alpha / (\nu - \eta)$ in evidence:

$$\left(\log \left(w_0^I / \underline{w} \right) - \frac{\alpha}{\nu - \eta} \right)^2 - \frac{2\alpha - \alpha^2}{(\nu - \eta)^2} = \left(\log \left(\underline{w}_0 / w_0^I \right) \right)^2 + \frac{2\alpha}{\nu - \eta} \underbrace{\left(\log \left(\underline{w}_0 / w_0^I \right) - \frac{1 - \alpha}{\nu - \eta} \right)}_{=T} \quad (79)$$

Notice that if T is positive, $[..]$ is positive, and the variance of log earnings increases with the minimum wage at \underline{w}_0 . Use the formula for α and define $x_0 = \underline{w}_0 / w_0^F > 1$ to rewrite T as:

$$T = \log(x_0 / (1 - \rho)) - \frac{x_0^\nu - 1}{(\nu - \eta)x_0^\nu + \eta} \quad (80)$$

where x_0 is determined by equation:

$$\frac{(1-\rho)^{\eta+1}}{\eta+1} + \frac{\eta}{\eta+1} x_0^{\eta+1} - x_0^\eta = 0 \quad (81)$$

Importantly, x_0 is pinned down independently of ν (inspection). To show that $T > 0$, I will show that it increases with $\nu > \eta$, and that at $\nu = \eta$, $T > 0$. Differentiate it with respect to ν :

$$\frac{\partial T}{\partial \nu} = \frac{x_0^\nu (x_0^\nu - 1 - \nu \log x_0)}{[(\nu - \eta)x_0^\nu + \eta]^2} > 0 \quad \Leftrightarrow \quad \frac{x_0^\nu - 1}{\nu} > \log x_0 \quad (82)$$

However, because $\nu > \eta$, the above condition is satisfied (as $\log x \leq x - 1$):

$$\frac{x_0^\nu - 1}{\nu} > \frac{x_0^\eta - 1}{\eta} \geq \log x_0 \quad (83)$$

At $\nu = \eta$ the term becomes:

$$T(\nu = \eta) = \log(x_0/(1-\rho)) - \frac{x_0^\eta - 1}{\eta} \quad (84)$$

I want to argue that for any $\rho > 0$, $T(\nu = \eta) > 0$. To do so, I show that at $\rho = 0$, $T(\nu = \eta) = 0$ and that $T(\nu = \eta)$ increases strictly with ρ . It is easy to see that when $\rho = 0$ implies $x_0 = 1$ and $T(\nu = \eta) = 0$.

The derivative is:

$$\frac{\partial T(\nu = \eta)}{\partial \rho} = \frac{1}{x_0} \left[\frac{\partial x_0}{\partial \rho} (1 - x_0^\eta) + \frac{x_0}{1 - \rho} \right] \quad (85)$$

Differentiate the equation that defines x_0 :

$$\frac{\partial x_0}{\partial \rho} = \frac{(1-\rho)^\eta}{\eta} \frac{1}{x_0^\eta} \frac{x_0}{x_0 - 1} \quad (86)$$

So:

$$\frac{\partial T(\nu = \eta)}{\partial \rho} = \frac{1}{1-\rho} - \frac{(1-\rho)^\eta}{\eta} \frac{1}{x_0 - 1} \frac{x_0^\eta - 1}{x_0^\eta} \quad (87)$$

Hence,

$$\frac{\partial T(\nu = \eta)}{\partial \rho} > 0 \quad \Leftrightarrow \quad \frac{(1-\rho)^{\eta+1}}{x_0^\eta} \frac{1}{\eta(x_0 - 1)} (x_0^\eta - 1) < 1 \quad (88)$$

But, from the definition of x_0 :

$$\frac{(1-\rho)^{\eta+1}}{\eta+1} + \frac{\eta}{\eta+1} x_0^{\eta+1} - x_0^\eta = 0 \quad \Rightarrow \quad \frac{(1-\rho)^{\eta+1}}{x_0^\eta} = 1 - \eta(x_0 - 1) \quad (89)$$

Substitute to find:

$$\frac{1-\eta(x_0-1)}{\eta(x_0-1)} (x_0^\eta - 1) < 1 \quad \Leftrightarrow \quad x_0^\eta < \frac{1}{1-\eta(x_0-1)} \quad (90)$$

which is true since (again from definition of x_0 , and $\rho > 0$):

$$x_0^\eta = \frac{(1-\rho)^{\eta+1}}{1-\eta(x_0-1)} < \frac{1}{1-\eta(x_0-1)} \quad (91)$$

F4 Proof of Proposition 4

Fix ℓ and solve for $\{l_h(z), w_h(z)\}$ by minimizing production costs:

$$C^{inf}(\ell; z) = \min_{\{l_h(z), w_h(z)\}} \left\{ \sum_h w_h(z) l_h(z) \right\} \quad (92)$$

$$\text{s.t. } \ell = \left[\sum_h \xi_h(z) l_h(z)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad l_h(z) = N_h \left[\frac{w_h(z)}{W_h} \right]^\eta \quad \forall h = 1, \dots, H \quad (93)$$

Solving the above problem I calculate the labor allocated at skill group h by firm z as a function of aggregates (due to the monopsonistic competition assumption) and ℓ :

$$l_h(z) = \frac{\xi_h(z)^{\frac{\eta\varepsilon}{\eta+\varepsilon}} \left(N_h^{1/\eta} / W_h \right)^{\frac{\eta\varepsilon}{\eta+\varepsilon}}}{\left[\sum_{h'} \xi_{h'}(z)^{\frac{\eta\varepsilon}{\eta+\varepsilon} \frac{1+\eta}{\eta}} \left(N_{h'}^{1/\eta} / W_{h'} \right)^{\frac{\eta\varepsilon}{\eta+\varepsilon} \frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}} \ell \quad (94)$$

I now turn to determining the scale of operation of the firms. Substitute for the cost function, which becomes linear in the cost index for firm z : $\mathbb{W}(z)\ell^{\frac{\eta+1}{\eta}}$. The profit maximization problem then reads:

$$\pi^{inf}(z) = \max_{\ell} \left\{ (1-\rho)z\ell - \mathbb{W}(z)\ell^{\frac{\eta+1}{\eta}} \right\} \quad (95)$$

and the resulting aggregate labor and profits are, respectively:

$$\ell(z) = \left[\frac{\eta}{\eta+1} (1-\rho) \frac{z}{\mathbb{W}(z)} \right]^\eta \quad (96)$$

$$\pi^{inf}(z) = \mathbb{W}(z)^{-\eta} \frac{\eta^\eta}{(\eta+1)^{\eta+1}} (1-\rho)^{\eta+1} z^{\eta+1} \quad (97)$$

It is easy to see that the formulas in the proposition are just algebraic manipulations of the results derived in this proof.

F5 Proof of Proposition 5

Substitute production function and CES labor aggregate constraints to find:

$$\pi^{form}(z) = \max_{\{w_h(z)\}_h} \left\{ z \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^\eta \frac{\varepsilon-1}{\varepsilon} w_h(z)^\eta \frac{\varepsilon-1}{\varepsilon} \right]^{\frac{1}{\varepsilon-1}} - (1+\tau) \sum_h \frac{N_h}{W_h^\eta} (1+\varsigma_h)^\eta w_h(z)^{\eta+1} \right\} \quad (98)$$

$$w_h(z) \geq \underline{w} \quad \forall h = 1, \dots, H \quad (99)$$

Let μ_h be the multiplier associated with the minimum wage constraint on skill h . The first order conditions are:

$$z \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^\eta \frac{\varepsilon-1}{\varepsilon} w_h(z)^\eta \frac{\varepsilon-1}{\varepsilon} \right]^{\frac{1}{\varepsilon-1}} \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^\eta \frac{\varepsilon-1}{\varepsilon} \eta w_h(z)^\eta \frac{\varepsilon-1}{\varepsilon} - 1 + \mu_h = (1+\tau) \frac{N_h}{W_h^\eta} (1+\varsigma_h)^\eta (\eta+1) w_h(z)^\eta \quad (100)$$

$$\mu_h (w_h(z) - \underline{w}) = 0, \quad \mu_h \geq 0 \quad \forall h = 1, \dots, H \quad (101)$$

If the wage offer is greater than the minimum wage then $\mu_h = 0$ and the first order condition becomes:

$$\frac{\eta}{\eta+1} \frac{z}{1+\tau} \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^\eta \frac{\varepsilon-1}{\varepsilon} w_h(z)^\eta \frac{\varepsilon-1}{\varepsilon} \right]^{\frac{1}{\varepsilon-1}} \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^{-\frac{\eta}{\varepsilon}} w_h(z)^{-\frac{\eta}{\varepsilon}-1} = 1 \quad (102)$$

Define $S \equiv \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1+\varsigma_h)^\eta \frac{\varepsilon-1}{\varepsilon} w_h(z)^\eta \frac{\varepsilon-1}{\varepsilon} \right]^{\frac{1}{\varepsilon-1}} \geq 0$ and notice that, for all $h = 1, \dots, H$,

optimal wages are a function of S :

$$w_h(z) = \max\{f_h(S; z), \underline{w}\}, \quad f_h(S; z) \equiv \left[\frac{\eta}{\eta+1} \frac{z}{1+\tau} \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{-\frac{1}{\varepsilon}} (1 + \varsigma_h)^{-\frac{\eta}{\varepsilon}} \right]^{\frac{\varepsilon}{\eta+\varepsilon}} S^{\frac{\varepsilon}{\eta+\varepsilon}} \quad (103)$$

Lastly, define the operator $T : \mathbb{R}_+ \mapsto \mathbb{R}_+$ as:

$$T(S) = \left[\sum_h \xi_h(z) \left(\frac{N_h}{W_h^\eta} \right)^{\frac{\varepsilon-1}{\varepsilon}} (1 + \varsigma_h)^{\eta \frac{\varepsilon-1}{\varepsilon}} \max\{f_h(S; z), \underline{w}\}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \quad (104)$$

where finding equilibrium wages boils down to finding a fixed point S^* such that $T(S^*) = S^*$, and then recovering wages through equation (103). Importantly, it is easy to see that $T(S)/S$ is continuous for all $S \geq 0$, it decreases with S , that $\lim_{S \rightarrow 0} T(S)/S = \infty$ and $\lim_{S \rightarrow \infty} T(S)/S = 0$. The above conditions imply that there exists a unique S^* such that $T(S^*) = S^*$, i.e., a unique solution to the problem of the formal firm.

Appendix G Walras' Law

This Appendix calculates the goods' market clearing.

Aggregate demand for goods read:

$$C = \sum_{h=1}^H \sum_{i=1}^{N_h} \sum_{j=1}^J Prob_{ihj} c_{ihj} \quad (105)$$

From the household problem $c_{ihj} = w_{hj}$ and conditional on h and the firm j the household is working for, all individual i 's are symmetric (that is: $Prob_{ihj} = Prob_{hj}$). This implies:

$$C = \sum_{h=1}^H \sum_{j=1}^J N_h Prob_{hj} w_{hj} \quad (106)$$

From the structure of the problem, in equilibrium I have that the labor demand for skill h by firm j equals the fraction of households of skill h that are choosing to work in that firm: $l_{hj} = N_h Prob_{hj}$, hence:

$$C = \sum_{j \in \mathcal{J}_{form}} \sum_{h=1}^H l_{hj} w_{hj} + \sum_{j \in \mathcal{J}_{inf}} \sum_{h=1}^H l_{hj} w_{hj} \quad (107)$$

where I also inverted the order of the summation and split firms into informal and formal sectors.

Each term in turn becomes:

$$\sum_{j \in \mathcal{J}_{form}} \sum_{h=1}^H l_{hj} w_{hj} = \sum_{j \in \mathcal{J}_{form}} q_j - \sum_{j \in \mathcal{J}_{form}} \pi_j^{form}, \quad \sum_{j \in \mathcal{J}_{inf}} \sum_{h=1}^H l_{hj} w_{hj} = (1 - \rho) \sum_{j \in \mathcal{J}_{inf}} q_j - \sum_{j \in \mathcal{J}_{inf}} \pi_j^{inf} \quad (108)$$

This implies that the goods market clearing condition:

$$C + \Pi^{form} + \Pi^{inf} + \rho Q^{inf} = \sum_{j=1}^J q_j = Q \quad (109)$$

which states that total production is split into consumption, profits for formal and informal firms, and government collection of revenue due to fiscalized informal units.

Appendix H SMM Estimation

This Appendix details the computation of the model.

For the SMM method, I will consider a version of the model with a discrete (but large) number of firms $n = 1, \dots, N = 300,000$. Before every simulation, I will draw productivities z_n from $F_z(z)$ and keep these draws constant.

To solve the model, I use the following algorithm:

1. Guess a vector of wage indices W_h^0 for $h = 1, \dots, H$
2. Calculate formal and informal profits, labor, and wages over a coarser grid on the productivity space:
 - 2.1. Informal profits $\pi^{inf}(z)$ from (??), with $W^{inf}(z)$ calculated from (??)
 - 2.2. Formal profits $\pi^{form}(z)$ from the fixed point (104), wages in (103), and profits in (??)
3. Interpolate profits, labor, and wages to find the respective variables for all firms $n = 1, \dots, N$
3. Update the vector of wage indices:

$$W_h^1 \equiv \left[\sum_{n=1}^N (1 + \varsigma_h(z_n))^{\eta} w_h(z_n)^{\eta} \right]^{\frac{1}{\eta}}, \quad \forall h = 1, \dots, H$$

4. Compare W_h^0 and W_h^1 for all h , update and iterate until convergence